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AUTHOR Ward, Barbara
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ABSTRACT

This is one of a series of reports summarizing the results of the second mathematics assessment conducted by the National Assessment of Educational Progress (NAEP). Nine, 13- and 17-year-olds were assessed during the 1977-78 school year. The assessment measured achievement in various content areas at four levels of cognitive processes (knowledge, skill, understanding and application). This report describes performance on application items. The report includes results for one-step word problems, problems about consumer situations, multi-step word problems, non-routine problems, and problems involving geometry, measurement, probability and statistics, graphs and tables, or reasoning making judgments. The report also includes group results, age level comparisons and observations on the results. The observations, consisting of interpretation, consideration of implications and recommendations, were made by a panel of persons active in the field. The panel felt that the results implied that problem solving is a major area of concern in mathematics education. Among the panel's seven recommendations were an expanded definition of what is "basic" modification of textbooks to include a greater variety of problem-solving tasks and more emphasis on the teaching of problem solving. (PK)

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MATHEMATICAL APPLICATIONS

SELECTED RESULTS FROM THE SECOND ASSESSMENT OF MATHEMATICS

Report No. 09-MA-03

by the
National Assessment of Educational Progress

Education Commission of the States
Suite 700, 1860 Lincoln Street
Denver, Colorado 80295

August 1979

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SE 028 820

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The cost figure cited above represents the total amount of money expended since late 1973 on assessments in art, career and occupational development, reading, writing, social studies/ citizenship, science, basic life skills, mathematics and consumerism, resulting, to date, in numerous reports, papers, articles, presentations and assessment materials, many of which are used in state and local assessment programs. A complete list of all such materials is available upon request.

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FOREWORD

When the U.S. Office of Education was chartered in 1867, one charge to its commissioners was to determine the nation's progress in education. The National Assessment of Educational Progress (NAEP) was initiated a century later to address, in a systematic way, that charge.

Each year since 1969, National Assessment has gathered information about levels of education achievement across the country and reported its findings to the nation. NAEP surveys the education attainments of 9-year-olds, 13-year-olds and 17-year-olds in 10 learning areas: art, career and occupational development, citizenship, literature, mathematics, music, reading, science, social studies and writing. Different learning areas are assessed every year, and all areas are periodically reassessed in order to measure possible changes in education achievement. National Assessment has interviewed and tested more than 810,000 young Americans since 1969.

Learning-area assessments evolve from a consensus process. Each assessment is the product of several years of work by a great many educators, scholars and lay persons from all over the nation. Initially, these people design objectives for each subject area, proposing general goals they feel Americans should be achieving in the course of their education. After careful reviews, these objectives are given to exercise (item) writers, whose task it is to create measurement instruments appropriate to the objectives.

When the exercises have passed extensive reviews by subject-matter specialists, measurement experts and lay persons, they are administered to probability samples. The people who compose these samples are chosen in such a way that the results of their assessment can be generalized to an entire national population. That is, on the basis of the performance of about 2,500 9-year-olds on a given exercise, we can make generalizations about the probable performance of all 9-year-olds in the nation.

After assessment data have been collected, scored and analyzed, National Assessment publishes reports to disseminate the results as widely as possible. Not all exercises are released for publication. Because NAEP will readminister some of the same exercises in the future to determine whether the performance level of Americans has increased, remained stable or decreased, it is essential that they not be released in order to preserve the integrity of the study.

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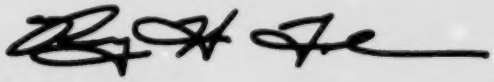
Mrs. Iris Carl, Dr. Thomas Carpenter, Dr. Robert Dilworth, Mr. Edgar Edwards Jr., and Dr. James Wilson contributed their time and knowledge interpreting the implications of the data. Their efforts helped greatly in putting the data into perspective.

The actual preparation of this report was a collaborative effort of the National Assessment staff. Special thanks must go to the following people: Ingrid Larsson and Charlotte Overbury for data processing support; Betty Homler for secretarial support; Ava Powell, Marci Reser and Paula Pitchford for production. Statistical analyses for this report were supervised by Donald Phillips. The report was written by Barbara Ward.



Roy H. Forbes
Director

INTRODUCTION

During the 1977–78 school year, the National Assessment of Educational Progress (NAEP) conducted its second mathematics assessment of the nation's 9-, 13- and 17-year-olds.[1] That assessment measured achievement in various content areas at four levels of cognitive processes – knowledge, skill, understanding and application. This report describes performance on application items.

The 1977–78 Mathematics Assessment

A two-dimensional matrix was used to guide the development of items for the 1977–78 assessment (*Mathematics Objectives, Second Assessment*, 1978, p. 17). Cognitive process levels formed one dimension of the matrix, and mathematical content areas, the other, as follows.

[1]The first NAEP assessment of mathematics was completed in 1972–73; changes in achievement between 1973 and 1978 are described in *Changes in Mathematical Achievement, 1973–78* (1979).

Items were developed to assess students' performance with respect to each of the "cells" of this matrix. Over 700 items were used.

This report focuses upon the fourth cognitive process level, mathematical application. Application refers to the use of the three lower cognitive levels – knowledge, skill and understanding – to solve problems. Students must use reasoning and judgment to select the appropriate procedures based on an understanding of the problem situation. They must also recall particular facts and definitions and have the skill to carry out appropriate algorithms.

The majority of items in the application section assessed performance on typical textbook problems. These covered a wide range of difficulty levels, from problems requiring only a single step or operation to problems requiring several steps. In addition to classic "number" problems, there were problems requiring

PROCESS	CONTENT: Numbers and Numeration	Variables and Relationships	Geometry (Shape, Size and Position)	Measurement	Other Topics
Mathematical knowledge					
Mathematical skill					
Mathematical understanding					
Mathematical application					

the application of concepts related to various topical areas such as geometry, measurement and probability. Other problems examined students' ability to use reasoning and judgment and their ability to solve nonroutine problems — that is, problems that would not typically be encountered in a mathematics textbook.

Measuring Achievement

The mathematics assessment was administered to a national probability sample of 9-year-olds, 13-year-olds and 17-year-olds who were still attending school at the time of the assessment. The 9-year-olds were assessed in January and February of 1978; the 13-year-olds in October through December of 1977; and the 17-year-olds in March and April of 1978. About three-fourths of the 9-year-olds in the NAEP sample were enrolled in the 4th grade, and about one-fourth were in the 3rd grade. Around three-fourths of the 13-year-olds were in the 8th grade, and most of the remainder were in the 7th grade. Approximately 75% of the 17-year-olds were in the 11th grade, about 15% were in the 10th grade and about 10% were in the 12th.

National Assessment does not report scores for individual students, but instead provides estimated percentages of correct responses for items. Thus, when a report indicates that "85% of the 17-year-olds gave a correct response," it means that an estimated 85% of the 17-year-olds would have given a correct response if all the 17-year-olds in schools across the country had been assessed. Therefore, it is not necessary for each student to respond to every item.

Each respondent completed only one exercise booklet of about 45 minutes in length. Between 2,100 and 2,500 students responded to each booklet. In 1977—78, there were 7 booklets for 9-year-olds, 11 for 13-year-olds and 12 for 17-year-olds. Approximately 17,000 9-year-olds, 27,000 13-year-olds and 27,000 17-year-olds participated in the 1977—78 mathematics assessment.

About half of the exercises in the second mathematics assessment were multiple-choice and about half were open-ended, meaning that students had to supply an answer rather than choose a response from a list of alternatives.

Responses to open-ended items were tabulated in various categories. These categories were defined based on results of exercise pretests conducted prior to the actual assessment. Scoring categories included likely errors and thus are useful in identifying frequently made mistakes. To minimize guessing, students were encouraged to write "I don't know" on the answer line on open-ended items or to select the "I don't know" option on multiple-choice items if they felt they did not know the answer to a question. In an effort to prevent reading ability from affecting performance, exercises were recorded on tape and read aloud to respondents as well as printed in the item booklets.

Additional Information on the Mathematics Assessment

The following reports, available from National Assessment, describe other aspects of the NAEP mathematics assessments: *Mathematical Knowledge and Skills* (1979) reports 1977—78 results on items assessing knowledge and skills; *Mathematical Understanding* (forthcoming) will describe students' performance in 1977—78 on exercises measuring understanding; *Changes in Mathematical Achievement, 1973—78* (1979) discusses changes in ability between the first and second assessments of mathematics. *Mathematics Objectives, Second Assessment* (1978) describes the objectives and the processes by content matrix used to structure the 1977—78 assessment.

Technical documentation of the 1977—78

assessment is presented in a released exercise set showing items released for publication, with national and selected group results; a microfiche data appendix providing additional group results for released exercises; and a yearbook describing assessment procedures. Reports and data volumes describing results from the 1972–73 mathematics assessment are also available.

CHAPTER 1

MATHEMATICAL APPLICATIONS

Exercises assessing students' performance in mathematical application required students to apply a sequence of processes in solving problems. Students had to recall relevant knowledge and determine needed skills based on an understanding of the situation and related concepts before applying these to the problem situation.

Most of the application items used in the NAEP mathematics assessment are of the type generally included in mathematics textbooks for practicing problem-solving skills. These "typical" textbook-type problems include one-step word problems, consumer problems (both one-step and multistep), multistep problems, and problems concerning particular content areas — geometry, measurement, graphs and tables, probability and statistics, and reasoning and logic. Some "nonroutine" problems, that is, problems of a type that would not generally be encountered in textbooks, were also included in the assessment.

Many of the items used to measure mathematical application have not been released for publication because they will be used to assess change in future assessments. Some of these unreleased items are described in terms of the task required, but the actual numbers used and specific details about the problem situations are not given.

One-Step Word Problems

Mathematical application exercises discussed in this section assessed students' ability to solve one-step word problems — those problems requiring the use of only one mathematical operation (addition, subtraction, multiplication or division) for their solution.

One-Step Problems Involving Addition of Whole Numbers

Addition is the first mathematical operation taught and the one with which primary-grade students are most familiar. Percentages answering correctly were higher for application items requiring addition than for any other operation. Simple one-step word problems using each of the four operations and employing whole numbers were assessed mainly at ages 9 and 13; there were very few of these problems for 17-year-olds.

A large percentage of 9- and 13-year-olds successfully solved the problem depicted in Exhibit 1 involving the addition of 2 two-digit whole numbers.

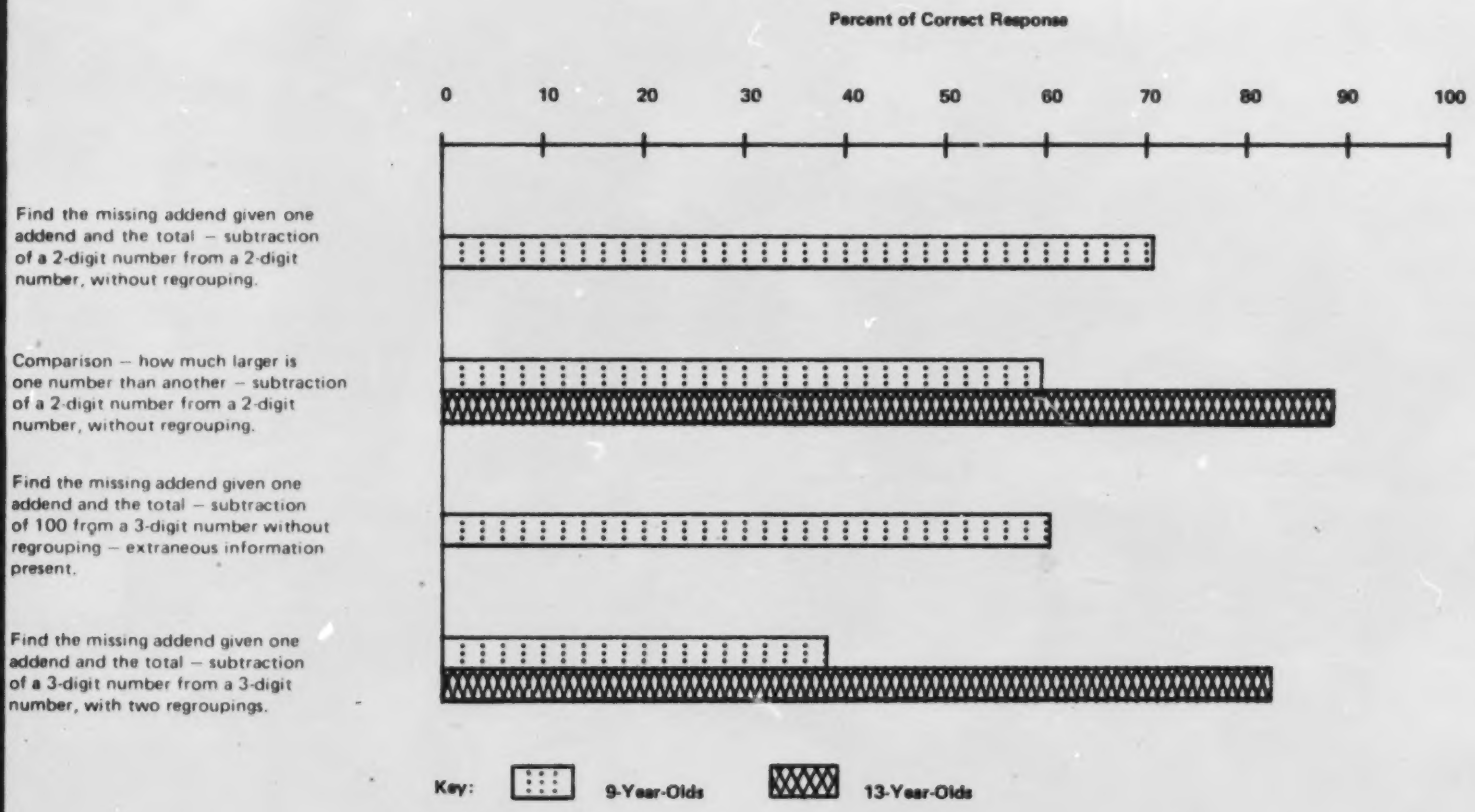
EXHIBIT 1. Percentages of Correct Response on One-Step Addition Item, Ages 9 and 13

Paul has 21 stamps in his collection. He buys 54 more from a stamp dealer. How many does he have after he buys them?

	Percent Correct	
	Age 9	Age 13
Respondents answering 75 or 75 stamps	82	96

To examine the relationship between computational skills and ability in solving application problems, the same numbers were occasionally presented both in a computational

EXHIBIT 2. Percentages of Correct Response on One-Step Problems Requiring Subtraction, Ages 9 and 13


Percent of Correct Response
0
10
20
30
40
50
60
70
80
90
100
Find the missing addend given one addend and the total – subtraction of a 2-digit number from a 2-digit number, without regrouping.
Comparison – how much larger is one number than another – subtraction of a 2-digit number from a 2-digit number, without regrouping.
Find the missing addend given one addend and the total – subtraction of 100 from a 3-digit number without regrouping – extraneous information present.
Find the missing addend given one addend and the total – subtraction of a 3-digit number from a 3-digit number, with two regroupings.
Key:
9-Year-Olds
13-Year-Olds

skill exercise, in which the operation to be used was indicated, and in a word problem, in which students had to determine the operation to be used. The same numbers used in the problem in Exhibit 1 were used in a computational skills item, 21 + 54 (listed vertically). Although a large percentage of 9-year-olds recognized that addition is the appropriate operation for the problem in the exhibit and carried out the operation correctly (82%), an even larger percentage of 9-year-olds (90%) were able to add the two numbers correctly when they were presented as a computational skill exercise. Thus, some of the 9-year-olds may have had difficulty deciding which operation to use when solving the word problem. Thirteen-year-olds' performance was similar on the computational and the word problem exercises — 98% and 96%, respectively.

Percentages of success for 9-year-olds were fairly high on two other word problems requiring addition. On one of them, requiring the addition of 4 one- or two-digit numbers, 79% gave the correct answer.

One-Step Problems Involving Subtraction of Whole Numbers

Four items assessed 9- and 13-year-olds' knowledge of one-step word problems requiring subtraction. The difficulty of the exercises varied. The required computations ranged from simple subtraction of two-digit numbers without regrouping (borrowing) to subtracting three-digit numbers with two regroupings. Performance declined as the subtraction exercises became more complex. Results for the four exercises are shown in Exhibit 2.

The first two exercises shown in Exhibit 2 involved the same task — subtracting a two-digit number from a two-digit number without regrouping. However, in the first problem the answer was a one-digit number and in the second, a two-digit number, meaning that students had to know two basic subtraction facts rather than one to solve the second problem. Results for a computational skill exercise using the same numbers and operation as the second problem were 15 percentage points higher than for the word problem for 9-year-olds and 8 percentage points higher for 13-year-olds.

The most difficult subtraction word problem for 9-year-olds called for subtracting 1 three-digit number from another and regrouping twice. On a subtraction computational exercise using the identical numbers, 50% of the 9-year-olds subtracted correctly, compared with 38% who answered the word problem correctly. The percentage of 13-year-olds answering the computational exercise correctly was only slightly higher than that for the word problem (85% compared with 82%).

One-Step Problems Involving Multiplication of Whole Numbers

Nine-year-olds' performance on application exercises requiring multiplication was generally lower than on word problems requiring addition or subtraction. For 13-year-olds, little difference in overall performance was seen on word problems using addition, subtraction or multiplication.

Exhibit 3 presents data for a simple multiplication word problem administered at all three age levels.

EXHIBIT 3. Percentages of Response on One-Step Multiplication Item, Ages 9, 13 and 17

Kate averages 10 miles per hour on her bike. At this rate how far will she travel in 5 hours?

	Percent Responding		
	Age 9	Age 13	Age 17
○ 2 miles	2	1	+
○ 5 miles	7	2	2
○ 15 miles	27	7	2
● 50 miles	54	88	94
○ More information is needed to solve this problem	4	2	1
○ I don't know.	6	+	+**

+Rounded percent less than one.

***Figures do not total 100% because of rounding and/or nonresponse.*

While most of the 13- and 17-year-olds answered this exercise correctly, only about half of the 9-year-olds did. Over 25% of the 9-year-olds simply added the two numbers given in the problem.

Performance of 13-year-olds was considerably lower on a problem giving extraneous numerical information (Exhibit 4). Many (nearly one-quarter) of them apparently multiplied all the numbers together.

EXHIBIT 4. Percentages of Response on One-Step Multiplication Item With Extraneous Information, Ages 9 and 13

One rabbit eats 2 pounds of food each week. There are 52 weeks in a year. How much food will 5 rabbits eat in one week?

	Percent Responding	
	Age 9	Age 13
○ 2 pounds	2	1
● 10 pounds	47	56
○ 52 pounds	16	5
○ 104 pounds	16	11
○ 520 pounds	12	23
○ I don't know.	6**	3**

***Figures do not total 100% because of rounding and/or nonresponse.*

In another exercise, 9- and 13-year-olds were given a word problem in which they had to multiply a two-digit number by a one-digit number. About 20% of the 9-year-olds and 77% of the 13-year-olds responded correctly. Twenty-seven percent of the 9-year-olds added the numbers. Results may have been lower for 9-year-olds on this exercise than for the two problems displayed previously because the multiplication involved required regrouping (carrying). Many 9-year-olds have had little or no experience with more complex multiplications involving regrouping. More 9- and 13-year-olds correctly multiplied the numbers used in the word problem discussed above when they were presented in a computational skill exercise than solved the word problem correctly. Thirty-four percent of the 9-year-olds and 84% of the 13-year-olds correctly multiplied the numbers, although only 20% and 77% of the 9- and 13-year-olds, respectively, gave the correct answer to the corresponding word problem.

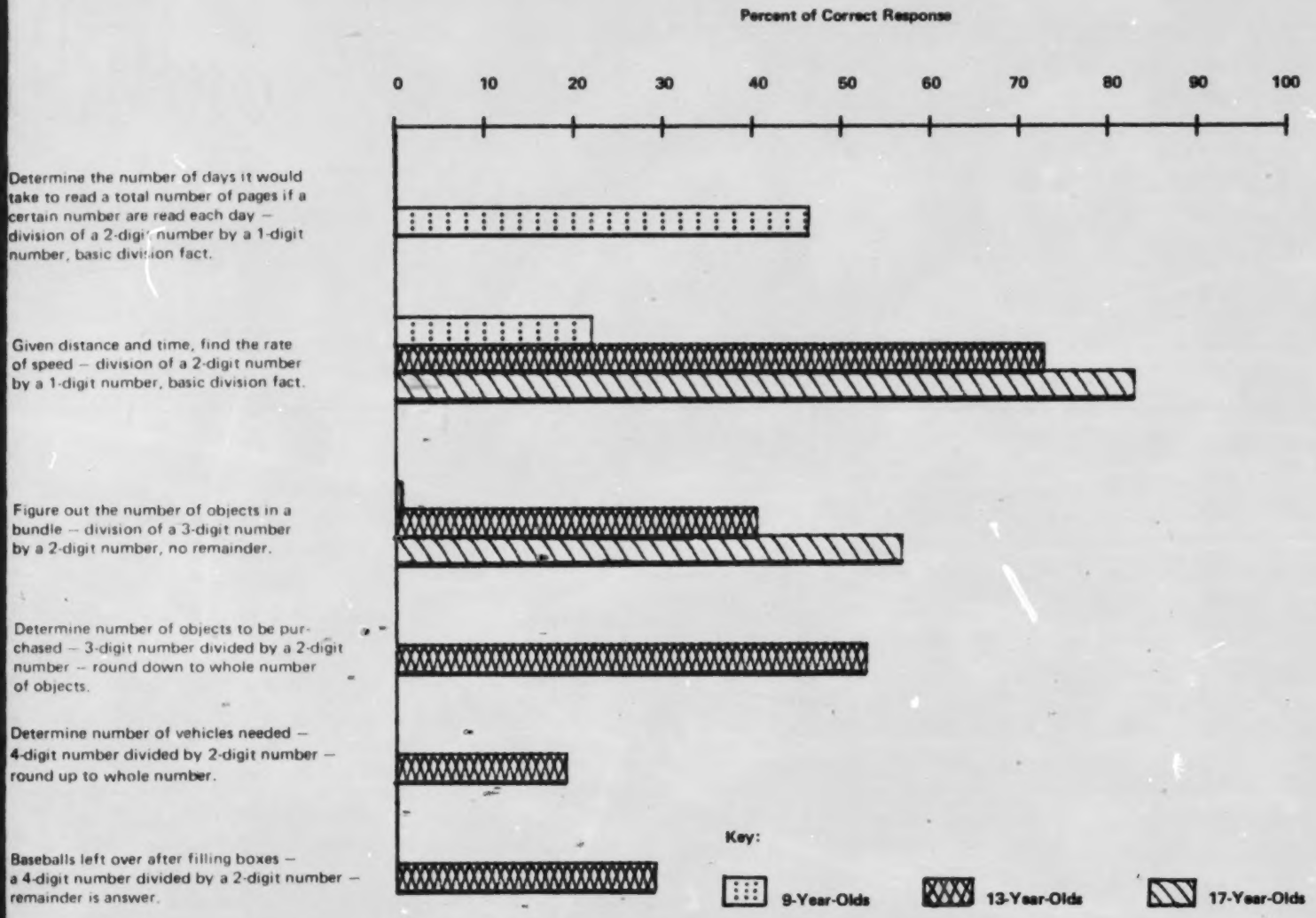
One-Step Problems Involving Division of Whole Numbers

Several word problems using division required only knowledge of basic division facts; others required skill in long division. As expected, performance was generally lower on the long division word problems, especially for 9-year-olds. Since 9-year-olds have had little experience with division, these results are not surprising. Results for the division word problems are shown in Exhibit 5.

The first two problems shown on the graph simply require knowledge of a basic division fact for their solution. Nearly half the 9-year-olds correctly determined how long it would take to read a certain number of pages if a specific number were read each day; 8% added the two numbers in the problem. Twenty-two percent of the 9-year-olds, 73% of the 13-year-olds and 83% of the 17-year-olds correctly used a basic division fact to compute rate of speed when distance and time were given. Incorrect response options chosen by 9-year-olds reflect their lack of experience with division. Twenty-three percent of them chose the sum of the two numbers, and another 25% chose the numeral given as the distance. This may have been an attractive option since it is the only numeral in the exercise — the time was expressed in words. Eleven percent at age 13 and 6% at age 17 also chose this option.

Long division is necessary for the remaining items depicted. In the first of these problems, students were told that a box held a certain number of bundles (two-digit) each containing the same number of objects and were given the total number of objects (three-digit) in the box. They were then asked to determine the number of objects in each bundle. Less than 1% of the 9-year-olds answered

EXHIBIT 5. Percentages of Correct Response for One-Step Problems Involving Division, Ages 9, 13 and 17


Percent of Correct Response
0
10
20
30
40
50
60
70
80
90
100
Determine the number of days it would take to read a total number of pages if a certain number are read each day -- division of a 2-digit number by a 1-digit number, basic division fact.
Given distance and time, find the rate of speed -- division of a 2-digit number by a 1-digit number, basic division fact.
Figure out the number of objects in a bundle -- division of a 3-digit number by a 2-digit number, no remainder.
Determine number of objects to be purchased -- 3-digit number divided by a 2-digit number -- round down to whole number of objects.
Determine number of vehicles needed -- 4-digit number divided by 2-digit number -- round up to whole number.
Baseballs left over after filling boxes -- a 4-digit number divided by a 2-digit number -- remainder is answer.
Key:
9-Year-Olds
13-Year-Olds
17-Year-Olds

correctly; 40% of the 13-year-olds and 57% of the 17-year-olds were successful. Most of the errors made by 13- and 17-year-olds were in the division operation itself — almost 20% at each age set up the problem correctly but failed to divide accurately.

The last three division problems required careful attention to the problem situation, since the exact answers to the needed computations are not the correct answers to the problems. These problems were administered only to 13-year-olds. Answers had to be rounded for two of these problems because parts of wholes are not appropriate answers for the problem situations.

More 13-year-olds were successful on the item involving money. This exercise gave a price for one object and asked how many of these objects could be purchased with a given sum of money. Since only a whole, not a part, object could be purchased, students had to round down to a whole number of objects. Fifty-three percent of the 13-year-olds answered correctly; another 15% gave the exact result of the long division without rounding.

Another item asked 13-year-olds to find out how many vehicles, each holding a certain number of persons, were needed to transport a group of people. Nearly 20% of them correctly rounded their answers upward to the next whole number, but almost 40% gave the exact quotient, apparently not thinking about the fact that their answers involved a fraction of a vehicle.

The item displayed in Exhibit 6 also requires careful reading because the question asks for the remainder obtained in the division operation, not the quotient itself. (For this exercise and for results on other open-ended exercises reported in the same manner, only the percentages of students giving the correct answer and a few other meaningful or interesting responses are reported; therefore, the percentages shown do not total 100%.)

Although only 29% of the 13-year-olds answered the problem correctly, another 22% realized that division is necessary to solve the problem. This 22% either set up the division problem correctly but did not solve it or did not read the problem carefully enough, giving as an answer the total number of boxes and the remainder when only the remainder was called for.

EXHIBIT 6. Percentages of Response for One-Step Division Item, Age 13

A man has 1,310 baseballs to pack in boxes which hold 24 baseballs each. How many baseballs will be left over after the man has filled as many boxes as he can?

	Percent Responding
Respondents answering 14*	29
Respondents answering 1,310 ÷ 24 or 54 R 14	22
Respondents answering 1,310 − 24 or 1,310 + 24	8
Respondents answering some remainder other than 14 on answer line	7

**Correct response.*

One-Step Word Problems Involving Fractions

Results for word problems involving fractions were higher for addition and subtraction problems than for those requiring multiplication and division. Most of the items in this section were administered only to 13- and 17-year-olds.

All three age groups were asked to add fractional quantities of liquids, pictured in two measuring cups. Students could simply visualize the correct answer from the picture rather than go through the process of adding fractions. Thirteen percent of the 9-year-olds, 56% of the 13-year-olds and 83% of the 17-year-olds added the fractional quantities correctly. However, when the same simple fractions were presented as a computational

skill item without a picture, only 2% at age 9, 35% at age 13 and 67% at age 17 added the fractions correctly. As seen in previous sections, students typically do better on computational skills exercises than on word problems using the same numbers. In this case, students did much better when they could visualize the solution than they did when the process of adding fractions was called for.

Exhibit 7 displays another word problem requiring addition of fractions. About the same percentage of 13- and 17-year-olds were successful on this problem, as on the previous one.

EXHIBIT 7. Percentages of Response for One-Step Problem Involving Addition of Fractions, Ages 13 and 17

To make a cake, you need 2-1/4 cups of sugar for the frosting and 1-1/2 cups of sugar for the cake. How many cups of sugar are needed for the cake and frosting together?

	Percent Responding Age 13	Age 17
○ 3-2/6 cups	31	10
○ 3-2/4 cups	10	7
● 3-3/4 cups	54	80
○ 3-2/2 cups	3	1
○ I don't know.	1**	1**

***Figures do not total 100% because of rounding and/or nonresponse.*

Similar percentages, 52% at age 13 and 77% at age 17, solved a word problem involving the subtraction of mixed numerals (whole numbers and fractions).

Two word problems required multiplication of the same simple fractions, but the problem situations given were different. Results were virtually identical for the two exercises. Eighteen percent of the 13-year-olds and 29% of the 17-year-olds correctly answered one problem, and 20% at age 13 and 31% at age 17 correctly answered the other. On a strictly computational multiplication item using the same fractions as the two word problems, substantially larger percentages — 69% at age 13 and 74% at age 17 — were successful.

Exhibit 8 displays 17-year-olds' performance on a one-step word problem requiring students to halve a fractional quantity. Twenty-nine percent gave the correct amount; an additional 22% gave an answer that was reasonable — between 1-1/2 and 2 — although not correct.

EXHIBIT 8. Percentage of Correct Response on One-Step Word Problem Involving Computation With Fractions, Age 17

A recipe for punch calls for 3-3/4 cups of pineapple juice for 10 people. How much pineapple juice should be used to make the same punch to serve five people?

	Percent Correct
Respondents answering 1-7/8	30

One-Step Word Problems Involving Percent

Results were varied on several items assessing use of percent in problem-solving situations. Slightly over one-fifth of the 13-year-olds and a little more than half of the 17-year-olds solved a problem asking them to find the percent one number is of another (Exhibit 9).

EXHIBIT 9. Percentages of Response on One-Step Item Involving Percent, Ages 13 and 17

A hockey team won five of the 20 games it played. What percent of the games did it win?

	Percent Responding Age 13	Age 17
Respondents answering 25% or 25*	21	52
Respondents answering 1/4 or .25 or 5/20	15	10

	Percent Responding	
	Age 13	Age 17
Respondents answering 4%	12	8
Respondents answering 15%	10	3

**Correct response.*

Students were more successful on a multiple-choice problem asking them to find a certain percent of a given number. Forty-six percent of the 13-year-olds and 69% of the 17-year-olds responded correctly.

In a more difficult word problem using percent, 13- and 17-year-olds were asked to find a percent of a weight given in pounds. To solve the problem, it is not necessary to convert to ounces, but the correct answer is less than one pound. Relatively few students responded correctly — 14% of the 13-year-olds and 40% of the 17-year-olds.

Problems About Consumer Situations

Many application problems included in the NAEP mathematics assessment focus upon consumer situations because these situations represent an immediate, practical application of mathematics and provide an indication of how well students do on problems that might actually confront them. For the purposes of this report, consumer problems are rather narrowly defined as those involving money or percent, primarily in purchasing situations. Other types of problems, such as using graphs and tables or finding perimeter, area or volume, which might be considered consumer related, are discussed in other sections of this report. Most of the problems described in this section are multistep (requiring more than one mathematical operation), although some one-step problems are included.

One-Step Consumer Problems

Some one-step problems required students to add prices found on a menu. Exhibit 10 shows one such problem, administered to all three age groups. Most of the 13- and 17-year-olds and slightly over half of the 9-year-olds were successful on this problem.

EXHIBIT 10. Percentages of Response on One-Step Consumer Item, Ages 9, 13 and 17

MENU

HAMBURGER .. $.85	MILK ..... $.20
HOT DOG ..... .70	SOFT DRINK ... .15
GRILLED CHEESE SANDWICH ..... .55	MILK SHAKE ... .45
	ICE CREAM .... .40
FRENCH FRIES . .40	

Sue had a hot dog, french fries, and milk. How much did she spend?

	Percent Responding		
	Age 9	Age 13	Age 17
○ $1.20	8	2	1
● $1.30	57	92	95
○ $1.40	9	2	2
○ $1.50	16	3	1
○ I don't know.	7**	+**	+**

+Rounded percent less than one.

***Figures do not total 100% because of rounding and/or nonresponse.*

Another problem involving a menu was administered only to 9-year-olds. Thirty-eight percent of the 9-year-olds selected the total asked for; about the same percentage, 37%, selected an answer indicating that they simply added the prices of everything on the menu.

A consumer-related, one-step division problem administered only to 17-year-olds proved quite difficult. The students were shown the following bill for electricity and asked how much the customer is paying per kilowatt hour.

Present Reading	Previous Reading	Consumed	Bill
1548 kw-hr	942 kw-hr	606 kw-hr	$9.09

Five percent of the 17-year-olds correctly responded 1.5¢ and an additional 4% gave 1.5 or 1-1/2 with incorrect or no units. Nineteen percent attempted to divide 606 by 9.09 instead of vice versa. Twenty-two percent simply responded "I don't know."

Multistep Consumer Problems

The multistep consumer problems reported in this section do not involve percent. The problems discussed here vary considerably in content and in mathematical tasks required, so it is difficult to make any general statements about performance.

Several items deal with totaling purchases and then determining amounts of change to be received from even dollar amounts. One problem involves finding the change one would receive from $1.00 when buying two different objects that together cost less than $1.00 — 37% at age 9 and 73% at age 13 selected the right answer. Twenty-eight percent of the 9-year-olds and 15% of the 13-year-olds selected an answer that suggests they had difficulty in regrouping (borrowing) when subtracting with zeros in the minuend (the number subtracted *from*).

A similar problem asked 13- and 17-year-olds to find the change one would receive from $20.00 when buying two objects costing less than $10.00 each. Sixty-two percent of the 13-year-olds and 77% of the 17-year-olds were successful. Eighteen percent of students at age 13 and 16% at age 17 selected answers indicating that they had difficulty subtracting with zeros in the minuend. Some students did not pay careful attention to the problem — 11% at age 13 and 4% at age 17 added the prices and gave that sum as the answer, not realizing that they were to find the change.

An item dealing with unit pricing asked teenagers to determine which was the better buy — to get four items at a designated price and get one free or to get five items at another price. To find the right answer, students had to perform two multiplication operations and then compare the results. Fifty-eight percent of the 13-year-olds and 70% of the 17-year-olds determined the better buy.

With the rising cost of fuel, gasoline mileage is of increasing interest to the consumer. On a problem concerning gas mileage, students were given two odometer readings and the number of gallons of gas consumed and then were asked to determine the average miles per gallon. Just over three-fifths of the 17-year-olds answered correctly, but only about one-fifth of the 13-year-olds did so. Concern over the task may increase as students reach driving age!

About half of the 17-year-olds correctly determined the week's earnings for a person who worked 40 hours at a specified hourly rate and a given number of additional hours at time and a half. Twenty percent of the 17-year-olds apparently mistook time and a half for half-time since they decreased the hourly rate by one-half rather than increased it. Another 17% of them added in the extra hours at the regular hourly rate, not at time and a half.

Relatively few (9%) of the 17-year-olds correctly calculated a taxi fare when a base fare and additional increments were given for fractions of a mile traveled and the distance traveled was expressed in decimals.

Consumer Problems Involving Percent

Percent is an important concept for the consumer. Interest rates, finance charges, sales tax and often sales prices are expressed in terms of percent. However, achievement levels on items involving percent generally were not high. Most of the consumer problems dealing with percent were administered only to 17-year-olds.

One item asked 17-year-olds to estimate a 15% tip on the price of a dinner for several people. Nearly one-fourth of the teenagers

named a tip that was between 14 and 16% of the total bill. Incorrect responses are interesting, as students tended to underestimate. Half the students left less than a 14% tip; about one-fourth left a tip that was 11—14% of the bill, and another quarter named a tip of less than 11% of the bill.

Thirteen- and 17-year-olds were asked to determine the savings on an item if the price is discounted 15%. Seventeen-year-olds were considerably more successful than 13-year-olds — 40% of them gave the correct savings, compared with only 10% of the 13-year-olds. Thirteen-year-olds tried two interesting approaches — 16% subtracted the percent from the item price, and 15% divided the item price by the percent.

In another problem, teenaged students were asked to find the percent of discount when the regular and sale prices were given (Exhibit 11). The majority of 13-year-olds (52%) and over a third of 17-year-olds inappropriately subtracted the difference in prices and claimed that the difference is the percent of discount.

EXHIBIT 11. Percentages of Response on Consumer Problem Involving Percent, Ages 13 and 17

REGULAR PRICE $48.00

SALE PRICE $36.00

What is the percent of discount?

	Percent Responding	
	Age 13	Age 17
○ 12	52	34
○ 20	14	14
● 25	18	36
○ 33-1/3	4	4
○ 75	3	3
○ I don't know.	9	7**

***Figures do not total 100% because of rounding and/or nonresponse.*

Other percent problems required more than one step for their solution. One such problem asked 17-year-olds to determine the depreciated value of a truck when given the original value and the percent of depreciation (Exhibit 12). One-quarter of the 17-year-olds gave the correct answer. About one-fourth evidently were able to compute percentages, but did not pay close enough attention to the problem and gave the amount of depreciation instead of the truck's value after depreciation.

EXHIBIT 12. Percentages of Response on Consumer Application Problem Involving Percent, Age 17

A trucking company estimates that one of its trucks depreciates at the rate of 25 percent of its original cost each year. If a truck originally cost $6,200, the value of the truck at the end of one year is:

	Percent Responding
Respondents answering $4,650 or 4,650*	25
Respondents answering $1,550, amount of depreciation	23

**Correct response.*

In another problem, 17-year-olds were asked to find the regular price of an appliance when the sale price and the percent off were given. Very few of the 17-year-olds (4%) answered this problem correctly. About one-fifth of the students subtracted the percent off from the sale price and then added the result to the sale price. This type of percentage problem — finding the whole when given a part and the percentage that part is of the whole — is one of the more difficult cases of percent. Items assessing skills in computing with percents show that many more students could take a percent of a number than could

determine the whole when given a part and the percentage the part is of the whole, or determine what percentage one number is of another (*Mathematical Knowledge and Skills*, 1979).

Installments, interest and finance charges represent a specialized use of percent. Students must understand these concepts as well as how to use percent in order to solve the following problems. Approximately one-third of the 17-year-olds correctly figured installment payments in the item shown in Exhibit 13. Slightly over one-fifth of the students either divided the amount of the loan by the number of months without adding the finance charge or correctly computed the finance charge but did not correctly determine the installments.

EXHIBIT 13. Percentages of Response on Consumer Application Item Involving Finance Charges, Age 17

Jerry bought an old Ford for $900. He paid $200 down and borrowed the rest. The total finance charge was 10% of the loan. He paid off the loan and finance charge in 10 equal installments. How much was each installment?

	Percent Responding
Respondents answering $77 or 77*	33
Respondents answering $70 (attempted $\{900-200\} \div 10$ or $\$700 \times .10$)	22

**Correct response.*

Twenty-nine percent of the 17-year-olds correctly calculated the total amount repaid on a loan when given the original amount of the loan and the percent of simple interest on the loan. Five percent set up the problem correctly but made inaccurate computations. Another 20% figured the amount of interest but did not add it to the original loan to obtain the total amount repaid.

Another problem dealing with finance charges does not involve percent. Students were given the price of an appliance, the down payment and the amount and duration of monthly payments. They were then asked to determine the total amount of finance charges. One-fifth of the 17-year-olds correctly determined the total amount. However, just under one-fifth multiplied the number of months times the amount of each installment, giving the total amount paid by installments rather than the amount of finance charges. Another 23% multiplied the installment amount times the number of installments and then added the down payment, giving the total cost of the refrigerator and neglecting to subtract the cash price to determine the finance charges.

Multistep Word Problems

The problems discussed in this section all require more than one operation for their solution. For many, algebraic solutions (i.e., the use of equations) either are required or are most efficient. Although some of these problems do involve money, they are not oriented toward real consumer situations.

EXHIBIT 14. Percentages of Response on Multistep Application Item, Ages 13 and 17

Ms. Baker has between $8,000 and $8,500 in her savings account. She wants to buy a new car that costs between $5,300 and $5,400. After she buys the car, how much money will Ms. Baker have in her savings account?

	Percent Responding	
	Age 13	Age 17
○ $2,700	4	2
○ $3,100	12	6
○ Between $2,700 and $3,100	59	64
● Between $2,600 and $3,200	21	27
○ I don't know.	2**	1

***Figures do not total 100% because of rounding and/or nonresponse.*

To solve the problem shown in Exhibit 14,

students had to determine which numbers to use when subtracting. The majority at each age evidently subtracted the smaller price from the smaller savings amount and the larger price from the larger savings amount, failing to realize that they had to account for the worst case — smallest savings account and most expensive car — and the best case — largest savings account and least expensive car.

Exhibit 15 displays a multistep problem involving distance, rate and time. Considerably more 17-year-olds than 13-year-olds were successful on this item. On an item involving ratios (Exhibit 16), percentages of success were similar to those in Exhibit 15; again, many more 17-year-olds than 13-year-olds answered correctly.

EXHIBIT 15. Percentages of Response on Multistep Application Item, Ages 13 and 17

A car traveled eight kilometers in five minutes. At this speed, how many KILOMETERS could it travel in one hour?

	Percent Responding	
	Age 13	Age 17
Respondents answering 96 km or 96*	28	56
Respondents attempting 8 × 5	7	3
Respondents attempting 60 × 8	16	10

**Correct response.*

EXHIBIT 16. Percentages of Correct Response on Multistep Application Item, Ages 13 and 17

Jerry's team won 3 out of 9 games. Joe's team won 1 out of 9 games. If both teams continue to win at this rate, and each team plays a total of 36 games in the season, how many more games will Jerry's team win than Joe's team?

	Percent Correct	
	Age 13	Age 17
Respondents answering 8	27	50

The remaining problems in this section are concerned with the application of algebraic concepts. One item asked 17-year-olds to set up and simplify an expression. Lengths of a triangle were given in terms of one side, which was defined as x, and students were to give the expression that represented the perimeter of the triangle. Twenty-three percent of the 17-year-olds were successful; about as many did not attempt to solve the problem but simply responded "I don't know."

Several items required students to substitute numbers into a given formula and solve for the missing element. About half of the 17-year-olds successfully solved for p in the formula $p = m/v$, given m and v, when p = density, m = mass and v = volume. The percentage of success (22%) was lower when students had to manipulate the formula to solve for the volume. In a similar problem using the formula $d = 16t^2$, 39% of the 17-year-olds correctly solved for the distance (d) when given the time (t). However, only 25% correctly solved for time (t) when the distance (d) was given.

The solution of the following typical textbook problem about the relationship of two numbers would be facilitated by the use of equations.

One number is 3 times as large as a second number. The sum of the two numbers is 72. What are the two numbers?

Answer 18 and 54

Twenty-eight percent of the students gave the correct numbers; another 8% set up an equation correctly but did not give the correct numerical answers. Twelve percent attempted to use only the numbers in the problem, dividing 72 by 3 to get 24; and 14% responded "I don't know." Exposure to algebra classes appears related to performance on this problem. Seventy-two percent of those who had completed mathematics courses beyond Algebra II and 46% of those who had completed Algebra II but no more ad-

vanced courses answered this problem right, compared with 28% of the nation as a whole.

Simultaneous equations are an efficient method of solving several of the multistep word problems. Most of these items are typical textbook problems involving mixtures. About one-third of the 13-year-olds and just over two-fifths of the 17-year-olds correctly solved a problem that, while simultaneous equations can be used, is readily answerable by inspection. Fewer 17-year-olds (20%) correctly solved a problem about a blend of two types of ground meat at different prices. Students had to determine the number of pounds of one type of meat given that the total number of pounds of the mixture costs a certain amount. This problem, although similar to the previous one, cannot be answered as easily by trial and error. It was solved correctly by 42% of those who had taken mathematics courses beyond Algebra II and by 25% of those who had completed Algebra II.

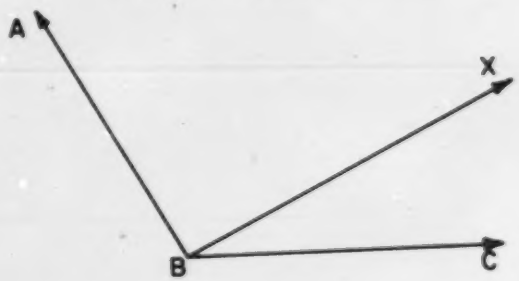
A typical textbook problem concerning age relationships of two children, which can most easily be solved by using simultaneous equations, was answered correctly by 14% at age 17. Those who had taken mathematics beyond Algebra II did substantially better (38% successful), and those who had completed Algebra II did somewhat better (18% correct) than the nation as a whole. Five percent of the 17-year-olds correctly computed the distance between two locations when given the total time of a round trip and the different rates of speed used each way. On this problem, only those who had taken courses beyond Algebra II performed significantly above the national level (16% correct).

Problems Involving Geometry

Some of the application problems included in the NAEP mathematics assessment relate to geometry. Approximately 46% of the 17-year-olds in the NAEP sample said that they had studied geometry for one school year; an additional 9% stated that they had had some geometry — a half year or less. Forty-two percent had never studied geometry. Many of the geometry application problems were administered to 13-year-olds even though some went beyond the informal geometry that 13-year-olds might be expected to have been taught in school. Some of the items in this section do require familiarity with geometric theorems; however, no formal geometric proofs were included.

A number of problems concern finding missing angles. For several items, students had to know that two angles with a common side can be added together; they also had to realize that one cannot assign exact measures to an angle by looking at it. On the item displayed in Exhibit 17, about one-fourth of the 13-year-olds and two-fifths of the 17-year-olds realized that there was not enough information to solve the problem. Many teenagers gave 90° as their answer — perhaps because ∠ABX looks like a 90° angle. On a similar problem, in which enough information was given to solve the problem, 42% of the

EXHIBIT 17. Percentages of Response on Application Item Involving Geometry, Ages 13 and 17



∠ ABC is obtuse. ∠ XBC is 40°. What is the measure of ∠ ABX?

	Percent Responding	
	Age 13	Age 17
○ 40°	6	2
○ 50°	12	7
○ 90°	35	30
○ 140°	13	10
● Not enough information given	24	41
○ I don't know.	10	10

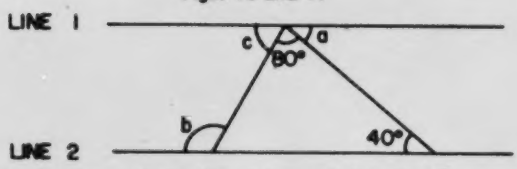
13-year-olds and 63% of the 17-year-olds were successful. Approximately 24% of the 13-year-olds and 11% of the 17-year-olds selected an answer indicating they assumed an angle that looked like a 90° angle is a 90° angle.

Students had to know that the interior angles of a triangle total 180° to find the missing angles of triangles. A knowledge item asking 17-year-olds to identify the equation representing the sum of the angles of a triangle indicates that 54% of them knew the correct total. Ten percent of the 13-year-olds and 40% of the 17-year-olds found a missing angle in a right triangle. This item is somewhat complicated by the fact that students had to know that a right angle equals 90° as well as know the sum of all the angles. Nearly 30% of the 13-year-olds felt that they had not been given enough information to solve the problem; only 8% of the 17-year-olds gave that response.

Knowledge of the equality of alternate interior angles is needed to answer the problems shown in Exhibit 18. (The angles labeled "a" and "40°" are alternate interior angles.) About one-third of the 13-year-olds and just over half of the 17-year-olds answered the first question correctly. The second question can be solved by using the information from the first question in conjunction with knowledge about alternate interior angles or by finding the missing angle of the triangle and using the knowledge that a straight angle measures 180°. The second question proved more difficult, especially for 13-year-olds.

Several questions concern the sum of interior angles for many-sided figures. About a third of the 13-year-olds computed the measure of one angle of a regular polygon when the sum of the interior angles was given. Twelve percent at age 13 and 29% at age 17 found the measure of one interior angle for a regular many-sided figure when the sum of the interior angles was not given. Only 6% at age 13 and 13% at age 17 determined the sum of the interior angles for an irregular, many-sided figure.

EXHIBIT 18. Percentages of Response to Two Application Problems Involving Geometry, Ages 13 and 17



In the figure above, line 1 is parallel to line 2.

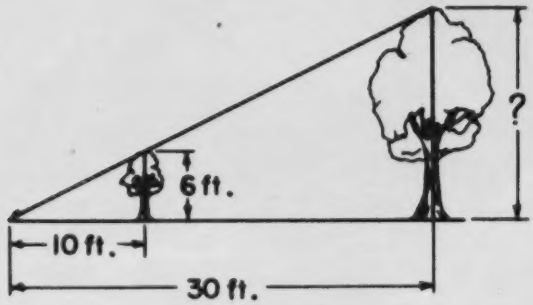
	Percent Responding	
	Age 13	Age 17
A. What is the measure of ∠ a?		
● 40°	34	55
○ 50°	6	5
○ 80°	27	13
○ It is equal to the measure of ∠ c	7	4
○ Not enough information given	16	15
○ I don't know.	9**	7**
B. What is the measure of ∠ b?		
○ 40°	18	7
○ 60°	18	14
● 120°	18	45
○ 140°	3	4
○ Not enough information given	30	19
○ I don't know.	12**	10**

***Figures do not total 100% because of rounding and/or nonresponse.*

Shown vertical angles (a ✕ b — a and b are vertical angles), 72% of the 13-year-olds and 81% of the 17-year-olds realized that vertical angles are equal. Ten percent at age 13 and 39% at age 17 used the information that a straight angle equals 180° to figure out the measure of a missing supplementary angle.

A number of problems required students to use proportions to find the length of a missing line segment. About one-third of the 13-year-olds and one-half of the 17-year-olds were successful on the problem displayed in Exhibit 19. In another problem, three parallel line segments cut by two transversals were displayed. Students had to realize that line segments cut off on one transversal are

EXHIBIT 19. Percentages of Response to Application Problem Involving Geometry, Ages 13 and 17



The picture above shows how José used a short tree to find the height of the tall tree. What answer should José get?

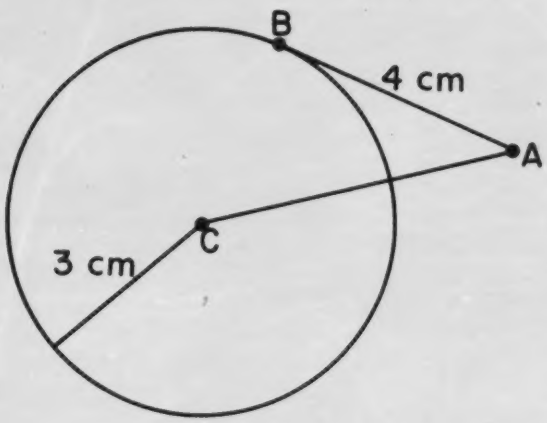
	Percent Responding	
	Age 13	Age 17
○ 12 feet	15	8
● 18 feet	36	50
○ 20 feet	9	9
○ 36 feet	15	11
○ 46 feet	17	9
○ I don't know.	6**	13

***Figures do not total 100% because of rounding and/or nonresponse.*

proportional to those cut off on the other to find a missing line segment. Fourteen percent of the 17-year-olds answered this item correctly. In an open-ended problem, 6% of the 17-year-olds gave the missing side of a triangle in a case in which a smaller triangle formed part of a similar larger one.

The Pythagorean theorem applies to a special case of triangles — right triangles. The problems in the assessment involving missing sides of right triangles can be solved by application of the Pythagorean theorem — the sum of the squares of the legs equals the square of the hypotenuse — or by recognition that 3-4-5 is a ratio for the sides of some right triangles. Nearly half the 17-year-olds correctly answered the item shown in Exhibit 20. To answer this question, students had to recog-

EXHIBIT 20. Percentages of Response to Application Problem Involving Geometry, Age 17



The tangent segment $\overline{AB}$ is 4 cm long. If the radius of the circle is 3 cm, how far is point A from the center of the circle?

	Percent Responding
○ 3 cm	3
○ 4 cm	5
● 5 cm	49
○ 6 cm	13
○ 7 cm	19
○ I don't know.	10**

***Figures do not total 100% because of rounding and/or nonresponse.*

nize that a radius is perpendicular to a tangent at the point of intersection; thus, $\triangle ABC$ is a right triangle. Nineteen percent at age 13 and 36% at age 17 solved a problem with a right triangle in which the sides of the figure shown, although proportionate to 3-4-5, are multiples of 100. Approximately 18% of the 17-year-olds solved a problem involving compass directions in which they first had to determine that a triangle was involved and then apply the appropriate procedures for working with the sides of right triangles.

Trigonometry is sometimes taught as part

of geometry courses; often it is included in Algebra II. Very few (6%) of the National Assessment 17-year-old respondents had taken a full-year course in trigonometry. An additional 12% had taken some trigonometry — a half year or less. Around 12% of the 17-year-olds correctly defined the sine, the cosine and the tangent when given a right triangle with sides labeled and the relevant angle identified. Eighteen percent of the 17-year-olds identified a specified trigonometric identity function from a list of options; 57% of them responded "I don't know." Seven percent were successful when asked to find a missing side of a triangle using trigonometry; 48% responded "I don't know" to this item.

As would be expected, 17-year-olds who had taken at least one year of geometry did considerably better on the geometry application problems than those who had not. For example, 65% of those who had taken a year of geometry answered the problem shown in Exhibit 17 correctly compared with 16% of those who had never taken geometry. Only those who had taken mathematics classes beyond Algebra II were above the national level in ability to answer the trigonometry questions.

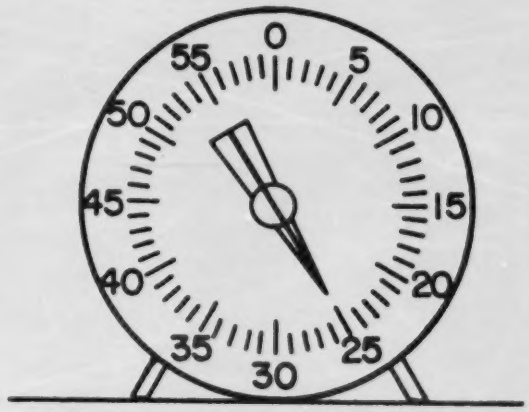
Problems Involving Measurement

Most of us are confronted with the necessity of solving problems involving measurement at one time or another in our lives. Such problems range from the fairly simple — what time to put the roast on — to the more complex — how much carpet is needed to cover a certain area, how much concrete will fill an allotted space.

Time is an important measurement concept for children to understand. About one-third of the 9-year-olds correctly calculated the length of time between two given times — simple subtraction without regrouping (borrowing) is required. Twenty-nine percent of the students at age 9 solved a problem requiring them to calculate a time eight hours after 10 p.m. This problem is complicated by the fact that the time goes past 12 and so one cannot simply add to solve the problem.

Exhibit 21 shows a problem about time administered to 17-year-olds; just over three-fifths solved it successfully.

EXHIBIT 21. Percentage of Correct Response on Application Item Involving Measurement, Age 17



Betty set the timer to run for 40 minutes. It had run for 15 minutes when she discovered it should have been set to run for 50 minutes. Where should she reset the timer now to correct the mistake?

	Percent Correct
Respondents answering 35 or 35 minutes	62

Seventeen-year-olds also responded to the following problem about cooking a roast:

A roast is to be cooked 20 minutes for each pound. If a roast weighing 11 pounds is to be done at 6:00 p.m., what time should it be put in the oven to cook?

Answer 2:20, 2:20 p.m.

About one-quarter of the 17-year-olds answered this problem correctly. For many 17-

year-olds, raw or burned dinners appear to be a real possibility.

Perimeter, area and volume, although concerned with geometric shapes, are essentially measurement concepts. Skill-level problems involving these concepts merely asked students to compute particular measurements; the problems discussed in this section are somewhat more complex.

In a skill-level exercise, about 40% of the 9-year-olds and 69% of the 13-year-olds calculated the "distance around" a rectangle when the rectangle with its dimensions was pictured. However, only 9% at age 9 and 31% at age 13 figured out how much fencing is needed to go around a rectangular garden when the garden was not pictured (Exhibit 22). Many students either added or multiplied the two given numbers.

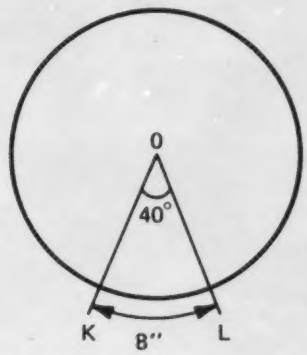
EXHIBIT 22. Percentages of Response on Application Problem Involving Measurement, Ages 9 and 13

Mr. Jones put a wire fence all the way around his rectangular garden. The garden is ten feet long and six feet wide. How many feet of fencing did he use?

	Percent Responding	
	Age 9	Age 13
○ 16 feet	58	38
○ 30 feet	6	3
● 32 feet	9	31
○ 36 feet	5	5
○ 60 feet	15	21
○ I don't know.	6**	2

***Figures do not total 100% because of rounding and/or nonresponse.*

About 20% of the 17-year-olds correctly stated that the circumference of the circle shown is 72 inches.



Almost 75% of the 17-year-olds, when shown a rectangular figure with its dimensions, correctly calculated the area of the rectangle. About three-fifths of them correctly found the length of a room, given the width (8 feet) and the area (96 square feet).

Forty-two percent of the 17-year-olds calculated the area of a square when shown the square and the dimension of one side; fewer (32%) calculated the area of the square when the perimeter was given but the figure itself was not shown.

Several problems about measuring area involve consumer situations. In one problem, 17-year-olds were told that a pound of seed covered a certain area and asked how many pounds of seed it would take to cover a larger area. Twenty-six percent of the 17-year-olds gave the right answer. However, only 13% of the 17-year-olds answered the following question correctly:

What is the price of a 12-foot by 15-foot piece of carpeting that sells for $7.00 per square yard?

Answer $140

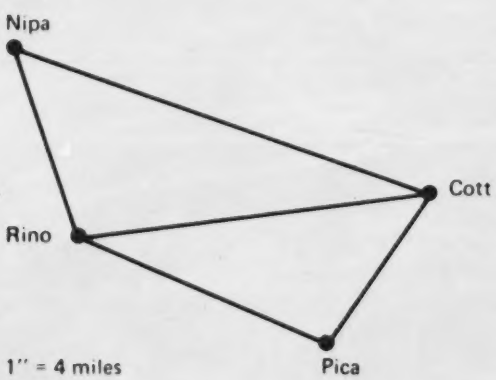
Approximately 26% of the students gave answers that indicate they either had difficulty in converting from square feet to square yards or did not realize they had to make the conversion. Some multiplied 12 × 15 × $7.00; others attempted (12 × 15 × $7.00) ÷ 3.

Given a figure that could be subdivided into two rectangles, and given its dimensions, only 16% of the 17-year-olds gave the correct area of the entire figure. However, approximately 36% were successful in determining the area remaining in a rectangle when a rectangular portion was removed from the center. Twelve percent of the 13-year-olds were successful at this task. Twenty-four percent at age 17 figured out the area of a rectangle when given the coordinates of the vertices on a coordinate plane, and 48% responded "I don't know" to this question.

Many 17-year-olds appear to have difficulty in calculating volume. On a skill item, 39% correctly calculated the volume of a rectangular solid, and an additional 6% used the right process but did not compute accurately or gave incorrect units. An application problem required students to calculate the volume of concrete needed for a pavement 30 feet by 20 feet by 4 inches. This exercise proved difficult for many students. Nine percent of the 17-year-olds gave the right answer; another 23% gave an answer indicating they had simply multiplied all three dimensions together without converting to a common unit.

EXHIBIT 23. Percentages of Response on Application Item Involving Measurement, Ages 9 and 13

In the map below, one inch represents four miles. How far is it from Rino to Cott? Use the ruler you have been given to find the answer.

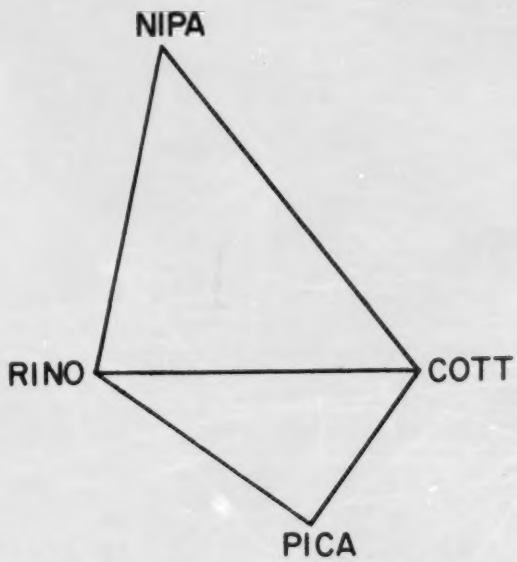


	Percent Responding	
	Age 9	Age 13
Respondents answering 8 miles to 8-1/2 miles*	25	67
Respondents giving the actual measured length of the line	33	10

**Correct response.*

Scale drawings are often used to depict measurements. To read a scale drawing, students must understand that a unit distance on

EXHIBIT 24. Percentages of Correct Response on Application Item Involving Measurement, Ages 13 and 17



The distance from Rino to Cott is 24 kilometers. Use this information together with the scale drawing and the ruler to find the distance from Rino to Pica.†

	Percent Correct	
	Age 13	Age 17
Respondents answering 18-22 kilometers	43	65

†Note that this drawing has been reduced in size for purposes of this report.

the drawing stands for some other unit distance. Exhibit 23 shows achievement levels on a problem involving a scale drawing. In this problem, students had to be able to physically use a ruler as well as apply the concept of scale.

The item shown in Exhibit 24 presents a more complex task. Students had to measure a line segment, determine the scale, measure a second line segment, and then use the scale to determine the actual distance corresponding to the second measurement. Another item gave a scale for a map and a distance on the map and asked students to give the corresponding real distance. The map itself was not shown. Fewer 17-year-olds (42%) were successful on this item than on the items depicted in Exhibit 24.

Problems Involving Probability and Statistics

Probability is the branch of mathematics concerned with situations in which the outcomes are uncertain, and it is also a basis for higher-level statistical concepts. The two subjects, probability and statistics, are often presented together.

By age 13, students would be expected to have some familiarity with elementary concepts of probability and statistics; thus, many of the items in this section were administered both to 13- and 17-year-olds.

Exhibit 25 shows a typical exercise in figuring probability. Slightly over a fourth of the 13-year-olds and slightly more than a third of the 17-year-olds were successful on this problem. Coin tossing is a classic context of problems in probability. Two problems about coin tosses are displayed in Exhibit 26. About one-third of the 17-year-olds answered the first problem correctly; the percentage for 13-year-olds was substantially lower. Performance of both groups was much higher on the second question. In fact, these percentages were higher than for any other questions in this section. These percentages may in part reflect the fact that the correct answer, one-half, is also the answer that is correct for the probability of a particular result when tossing one coin. On a similar

EXHIBIT 25. Percentages of Correct Response on Application Item Involving Probability, Ages 13 and 17

2, 3, 4, 4, 5, 6, 8, 8, 9, 10

For a party game each number shown above was painted on a different Ping-Pong ball, and the balls were thoroughly mixed up in a bowl. If a ball is picked from the bowl by a blindfolded person, what is the probability that the ball will have a 4 on it?

	Percent Correct	
	Age 13	Age 17
Respondents answering 1/5, 2/10 or 1 out of 5	28	35

EXHIBIT 26. Percentages of Response on Application Item Involving Probability, Ages 13 and 17

A. Mike has 2 quarters. What is the probability he will get 2 heads when he flips them?

	Percent Responding	
	Age 13	Age 17
● 1/4	18	33
○ 1/3	5	3
○ 1/2	58	50
○ 2/3	8	4
○ 3/4	3	2
○ I don't know.	8	8

B. What is the probability he will get one head and one tail when he flips them?

○ 1/4	11	12
○ 1/3	6	3
● 1/2	60	69
○ 2/3	7	4
○ 3/4	8	6
○ I don't know.	8	6

problem involving three coins, 5% of the 17-year-olds replied correctly.

Combinations and permutations are often used in calculating probabilities. Combinations involve subsets of particular objects or events out of a total set of possible objects or events without regard to order. An arrangement of a set of objects in a certain order is called a permutation of that set.

EXHIBIT 27. Percentages of Response on Application Problem Involving Combinations, Ages 9 and 13

Dale had dark slacks and white slacks.

Dale also had 3 different shirts. They were plaid, plain, and striped. How many different outfits (slacks and shirt) could Dale wear?

	Percent Responding	
	Age 9	Age 13
○ 2	59	16
○ 3	9	5
○ 5	16	10
● 6	13	68
○ I don't know.	2**	+**

+Rounded percent less than one.

***Figures do not total 100% because of rounding and/or nonresponse.*

About two-thirds of the 13-year-olds solved a simple combination problem (Exhibit 27). Considerably fewer 9-year-olds were successful. Forty-two percent of the 9-year-olds and 68% of the 13-year-olds successfully gave the number of ways a few digits could be ordered when some digits were repeated. About one-fifth of the 9-year-olds and two-fifths of the 13-year-olds figured out the number of contacts that would be made if each child in a group contacted each other child in the group once. The total group was fairly small so that students could figure out the answer without using a computational algorithm.

Several problems required students to find the number of permutations for a certain number of objects from a total set of objects. About 30% of the 17-year-olds were successful when the objects were pictured and the item format was multiple-choice. However, only 8% of the 17-year-olds and 4% of the 13-year-olds were successful when the same numbers were used in a multiple-choice problem in which the objects were not pictured. About the same percentages (8% at age 17 and 3% at age 13) were successful when the same number of objects were presented in an open-ended item without pictures. Relatively few 17-year-olds were successful in the two problems shown in Exhibit 28.

Knowledge of statistics is useful in summarizing data so that large amounts of information can be more easily comprehended. The study of statistics is often separated into descriptive statistics, dealing with ways of presenting numerical data, and inferential statistics, involving drawing conclusions about a large population from information obtained from a small subset or sample of the larger population. Most of the statistics taught in elementary and secondary mathematics classes is descriptive in nature.

Some of the skills and understanding problems tapped ability in computing averages and in understanding statistical terminology. Six percent of the 9-year-olds, 51% of the 13-year-olds and 71% of the 17-year-olds computed the "average" of several one- and two-digit numbers. However, when given the same numbers, 6% and 12% of the 13- and

EXHIBIT 28. Percentages of Response on Two Application Items Involving Combinations, Age 17

A combination lock on a trunk has three dials, one showing all 26 letters of the alphabet and the other two showing the 10 digits 0 to 9. Assuming that a combination uses a setting on all three dials, how many different combinations are possible?

	Percent Responding
Respondents answering 2,600*	15
Respondents answering 260	10

If each of the six teams in a bowling league plays every other team ONCE, how many games will be played in all?

	Percent Responding
Respondents answering 15*	5
Respondents answering 30	13
Respondents answering 36	27

**Correct responses.*

17-year-olds, respectively, correctly identified the "mean." Forty-two percent at age 13 and 43% at age 17 identified the median, and 24% at age 13 and 26% at age 17 correctly selected the mode. Familiarity with these terms does not appear to increase greatly between ages 13 and 17. Average is obviously a much more familiar term than mean.

On an application problem, 46% of the 13-year-olds and 69% of the 17-year-olds used the concept of average to figure out how many points a player would make in a number of games, assuming a constant scoring average per game. Fewer students (16% at age 13 and 26% at age 17) figured out the missing number of a set needed to obtain a certain average based on all the other numbers in the set. Two other problems involve weighted averages; percentages of success were relatively low on these items. Seven percent of the 13-year-olds and 17% of the 17-year-olds figured out the average cost of candy when candy costing different prices was mixed together; 13% of the 17-year-olds determined the average wage for a group of workers paid at several different rates.

Problems Involving Graphs and Tables

Graphs and tables are used in the popular press as well as in more technical publications in order to present a great deal of information concisely. The citizen and consumer must be able to read and interpret data presented in such forms in order to make informed decisions.

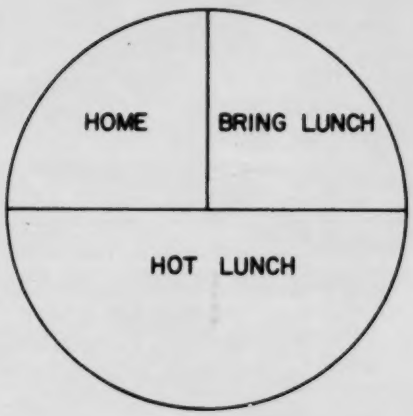
Exhibit 29 shows performance of 9-, 13- and 17-year-olds on problems dealing with a circle graph. To solve these problems, students had to realize that fractional parts of the circle were equivalent to fractional parts of the total number of students being considered. Performance on these questions increased from age 9 to 13 to 17.

Thirteen- and 17-year-olds responded to a series of questions about information in a table comparing population sizes of two cities in different years. Many teenagers (70% of the 13-year-olds and 87% of the 17-year-olds) correctly compared relative populations of the two cities, and somewhat fewer (62% at age 13 and 78% at age 17) correctly compared the population of the same city in different years. The students had more difficulty in projecting to the future using past trends — 35% of the 13-year-olds and 60% of the 17-year-olds accomplished this task correctly.

One-fourth of the 13-year-olds and one-half of the 17-year-olds found the nutritive values of certain foods on a chart and then employed that information in calculating nutritive values of a meal. About 10% at age 17 correctly used a table about injuries related to auto accidents showing pedestrian injuries and total number of injuries in various age groups to determine which group had the highest

EXHIBIT 29. Percentages of Response on Application Item Involving a Graph, Ages 9, 13 and 17

KINDS OF LUNCHES STUDENTS EAT AT WARREN JUNIOR HIGH SCHOOL



	Percent Responding		
	Age 9	Age 13	Age 17
A. If 400 students eat lunch, about how many go home for lunch?			
○ 25	27	12	6
● 100	29	60	72
○ 200	20	18	16
○ 300	4	2	1
○ 400	5	2	2
○ I don't know.	15	6	2**
B. If 400 students eat lunch, what is the total number of students that either bring lunch or eat a hot lunch?†			
○ 75	21	9	4
○ 100	14	6	4
○ 200	20	14	10
● 300	22	51	67
○ 400	9	12	9
○ I don't know.	13**	7**	4**

†Wording for 9-year-olds differed slightly on this question. Nine-year-olds were asked, "If 400 students eat lunch, about how many bring lunch or eat a hot lunch?"

***Figures do not total 100% because of rounding.*

	Percent Responding	
	Age 13	Age 17
C. If you asked 30 students "Are you going home for lunch?" about how many would you expect to say "Yes"? §		
○ 0	6	4
● 8	44	60
○ 15	30	20
○ 30	11	8
○ I don't know.	8**	7**

§ This question was administered only to 13- and 17-year-olds.

***Figures do not total 100% because of rounding and/or nonresponse.*

number of pedestrian injuries per total injuries.

Problems Involving Reasoning and Making Judgments

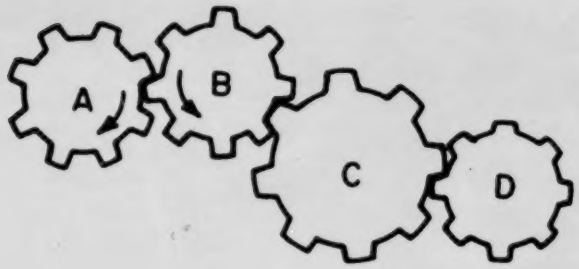
Reasoning and making judgments are essential elements in problem solving. A number of problems measured ability in reasoning, using logic and applying concepts of probability to make judgments. These problems generally did not require the use of computation to find the answers.

As shown in Exhibit 30, 73% of the 13-year-olds and 81% of the 17-year-olds determined the relationship of the gears pictured.

Several problems concern the relationship between statements. Most students (82%, 93% and 96% of the 9-, 13- and 17-year-olds, respectively) realized that if a particular object has two characteristics (for example, a box is red and green), then an object having only one of those characteristics (a box that is red and blue) cannot be the same as the original object. Fewer students — 39% at age 9, 71% at age 13 and 86% at age 17 — realized that an object with the same two characteristics as the first object described might be, but is not necessarily, the same object.

EXHIBIT 30. Percentages of Response on Application Problem Involving Reasoning and Judgment, Ages 13 and 17

Gear A is rotating clockwise. Gear B is rotating counterclockwise. Which way is Gear D rotating?



	Percent Responding	
	Age 13	Age 17
○ Clockwise	16	15
● Counterclockwise	73	81
○ Can't tell	10	3
○ I don't know.	1	+**

+Rounded percent less than one.
***Figures do not total 100% because of rounding and/or nonresponse.*

The same pattern is apparent in another set of questions. Given that all members of a group have one characteristic (e.g., all basketball players are tall), 67% at age 9, 64% at age 13 and 72% at age 17 realized that someone with an opposite characteristic (a short person) cannot be part of the group. Unlike the previous questions, these items gave students the option of indicating that there was not enough information to solve the problem, and many students at each age chose this option. Twenty-five percent of the 9-year-olds, 51% of the 13-year-olds and 58% of the 17-year-olds correctly stated that there was not enough information to know if a person with the characteristic of a group is a member of the group (e.g., a tall person might or might not be a basketball player).

Given a statement of the form if *a* then *b*, 30% of the 13-year-olds and 51% of the 17-year-olds realized that if *a* then not *b* is the only situation from a list of options that cannot be true.

Several items required probabilistic judgments or asked students to determine the likelihood of events. About a third of the 9-year-olds, just over three-fifths of the 13-year-olds and just over three-fourths of the 17-year-olds realized that, in the situation described below, Chris and Ellen would never be on the same team.

Two team captains take turns choosing players for their team. Ellen is always chosen first. Chris is always chosen second. If Ellen and Chris are never captains, how often do they play on the same team?

Given a group of people in which different persons performed different activities, 47% of the 9-year-olds and 73% of the 13-year-olds correctly inferred that if they did not know a particular person's activity, it was most likely that a person would be doing the activity participated in by the largest number of group members. Eighty-four percent of the 13-year-olds and 92% of the 17-year-olds realized that odds of getting heads or tails are equal for one flip of a coin. However, in a case in which losing and winning the toss produced different monetary rewards, 30% of the 13-year-olds and 33% of the 17-year-olds realized that, over a series of events, equal probabilities would not produce equal monetary results. Fifty-seven percent at age 13 and 56% at age 17 wrongly replied that the chances of winning or losing money are equal, not realizing that different monetary rewards would alter the situation. Relatively few teenagers (13% at age 13 and 21% at age 17) figured out what sum would be most likely to come up on two spinners.

Nonroutine Problems

Most of the problems in the preceding sections are those that are typically found in mathematics textbooks; most students would have some experience in solving similar problems. The problems presented in this section

are generally beyond the scope of classic textbook application problems and require application of knowledge, skills and understanding to somewhat unfamiliar situations. In these problems, students had to think through the logic of their answers and, generally, could not simply apply a formula. Problems in this section are quite varied, and percentages of success appear highly dependent on the nature of the problem.

Exhibit 31 shows one item classified as a "nonroutine" problem. This problem could be solved either by making a table of weekly increments or by setting up an algebraic expression. Performance was not particularly high on this item. Another item described the relationship of three objects in a line. The distances between A and B and B and C were given, and students had to determine the relationship between A and C. Many students did not realize that C could be located on either side of B — making two distances correct answers. Fourteen percent of the 9-year-olds, 26% of the 13-year-olds and 44% of the 17-year-olds answered this problem correctly.

EXHIBIT 31. Percentages of Correct Response on a Nonroutine Application Problem, Ages 9, 13 and 17

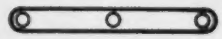
Right now Bob has $12 in the bank and Carol has $26 in the bank. From now on each will save $1 every week. How much money will Bob have saved when he has half as much as Carol?

	Percent Correct		
	Age 9	Age 13	Age 17
Respondents answering $14	12	20	32

To solve the problem displayed in Exhibit 32, students had to determine how many of the most frequently used parts of the object were needed for each object and then divide that number into the total number available. About one-fourth of the 13-year-olds answered correctly; very few 9-year-olds were successful. Twenty percent of the 9-year-olds multiplied 60 × 3, resulting in an answer of 180.

EXHIBIT 32. Percentages of Correct Response on a Nonroutine Application Problem, Ages 9 and 13

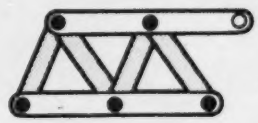
Mike's building set has
60 long pieces



60 short pieces

and 60 nuts with bolts.

How many of these can he make?



	Percent Correct	
	Age 9	Age 13
Respondents answering 12	3	24

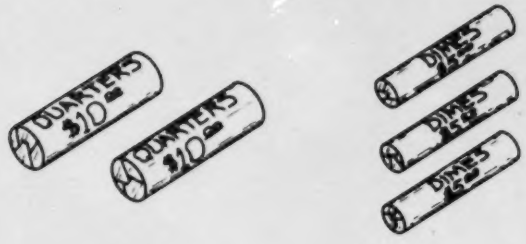
Nearly one-third of the 13-year-olds and about two-thirds of the 17-year-olds realized that if votes are distributed over a number of candidates, and the winner is the candidate with the most votes, the smallest number of votes the winning candidate can obtain is (total number of votes ÷ number of candidates) + 1.

Several nonroutine problems were administered only to 17-year-olds. In the problem shown in Exhibit 33, 17-year-olds had to figure out the total number of quarters and dimes available, determine the number of each needed for a set of 45 cents, and then figure out whether quarters or dimes would run out first. Forty-two percent of them were successful.

Thirty-five percent of the 17-year-olds correctly found the limit of a simple infinite series. Thirty-two percent of them correctly selected the number from a list of options that could be written in a given algebraic form. Only 8% at age 17 found the length of the diagonal of a rectangular solid when given the dimensions of its edges.

EXHIBIT 33. Percentages of Response on a Nonroutine Application Problem, Age 17

Juan's mother has three five-dollar rolls of dimes and two ten-dollar rolls of quarters to use for Juan's school lunch.



If Juan takes exactly forty-five cents to school every day for his lunch, which of the following statements is true?

	Percent Responding
○ He uses all of the quarters before all of the dimes.	17
● He uses all of the dimes before all of the quarters.	42
○ He spends all of both coins at the same time.	36
○ I don't know.	4**

***Figures do not total 100% because of rounding and/or nonresponse.*

CHAPTER 2

GROUP RESULTS AND AGE-LEVEL COMPARISONS

To examine the relationships between the characteristics of students and mathematics achievement, National Assessment analyzes results for various groups within the national population. Some of these groups are defined by students' backgrounds or personal characteristics; others are related to school experiences.

Group Results

Groups related to students' background characteristics are defined based on region of the country, sex, race, level of parents' education, and type and size of community lived in. Groups based on school experience are defined by level in school and, for 17-year-olds, types of mathematics courses taken. Definitions of groups reported on appear in Appendix A.

National Assessment found that similar patterns of group results on the application items prevailed across age groups — the same groups were generally either above or below national levels, although in some cases tendencies became more marked for the older students.

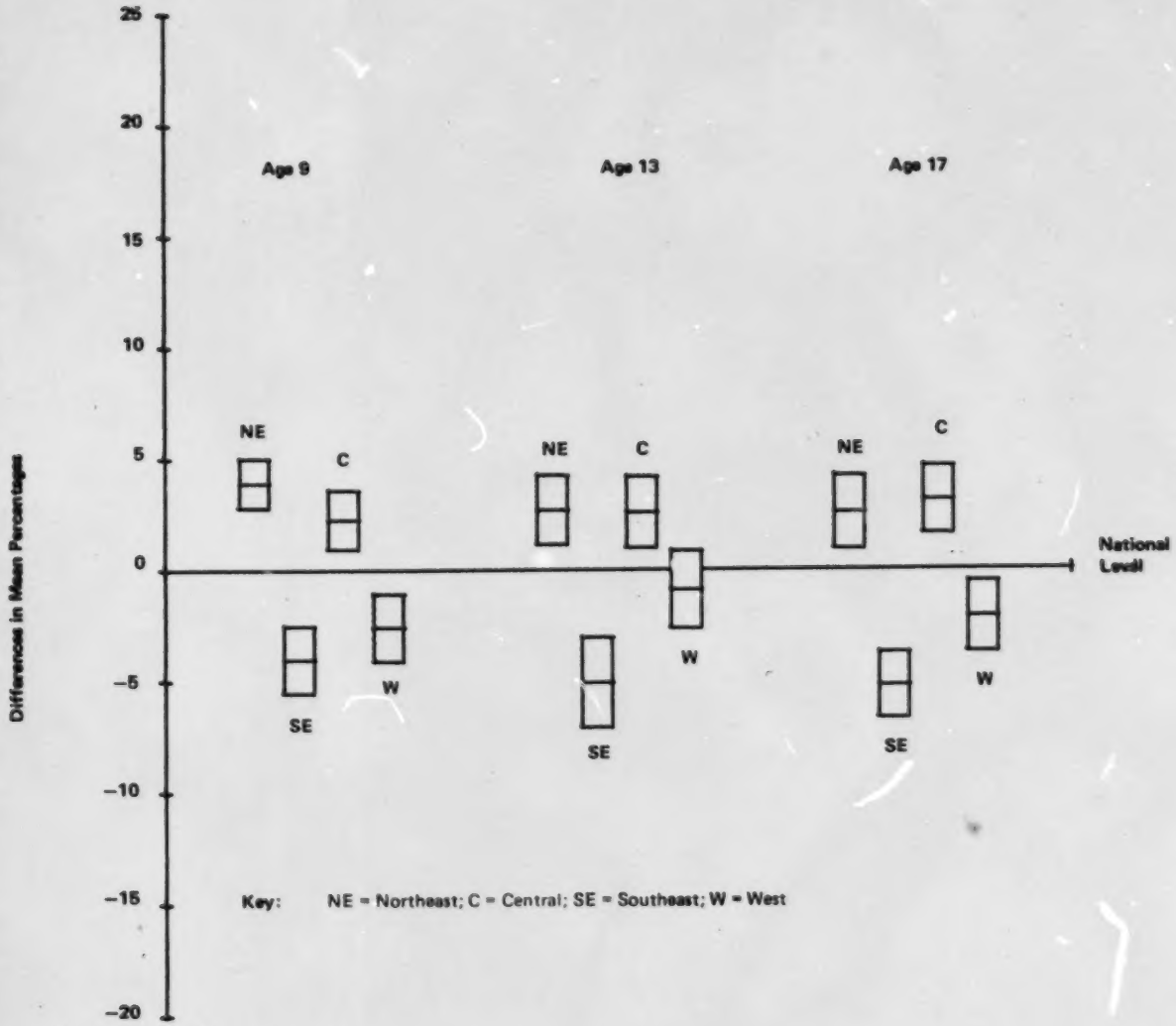
Description of the Data

Group performance is described in terms of average performance on a particular set of exercises compared with the national performance level on that same set. Average performances have been computed for all application items and for the various subsets of items discussed in the previous chapter. Exhibits 34—40 and 42 display differences between group and national performance for the various types of groups on the entire set of mathematics application items. In these exhibits, the zero line represents the national performance level. The short, horizontal solid line for each group represents the difference between that group's mean and that of the nation. A positive difference (i.e., a line above the zero line) indicates that the group performed above the national mean, and a negative difference (i.e., a line below the zero line), below it. The bar surrounding each mean represents two standard errors above and below the mean achievement for the group.

A standard error of the sample mean is an estimate of the sampling variability among the means of all possible samples; it is used to estimate the precision of the mean obtained in a particular sample. Intervals of two standard errors below to two standard errors above a particular statistic would include the average of the statistic in approximately 95% of all possible samples of the same size. An interval computed in this way is a 95% confidence interval. If the standard error bar for a particular group mean does not cross the national line, we can be 95% confident that the group mean is different from the national mean.

It should be remembered that differences in achievement levels cannot be ascribed solely to membership in a particular group being discussed. Research has shown that any number of socioeconomic, school-related and environmental factors are related to achievement, and, caution must be taken not to attribute a cause-and-effect relationship between membership in a specific group and performance.

EXHIBIT 34. Differences in Mean Percentages Between Regional Groups and the Nation on Application Items, Ages 9, 13 and 17



Background Variables

The following sections describe patterns of results for the various groups. Appendix B presents average group performance relative to the national level for each group for the entire set of application items and for various subsets of these items.

Region. As seen in Exhibit 34, the relative positions of regional achievements are fairly constant across age levels. The Northeastern (NE) and Central (C) regions were above the nation for all three age groups, and the Southeastern (SE) region was below the nation. The West (W) fell below the national level at ages 9 and 17, but at age 13, Western students performed about the same as the nation. The same patterns are evident for most of the subsets of application items discussed in this report.

Sex. Differences in males' (M) and females' (F) performance were not constant across ages (Exhibit 35). At age 9, there was almost no difference in males' and females' achievement; by age 13, a slight disparity favoring males was observed; and by age 17, the gap in achievement had widened considerably. Male 13-year-olds did better than their female counterparts on multistep problems, problems involving graphs and tables, and geometry. On the average, males' and females' achievement at age 13 was the same on one-step word problems, consumer problems, problems involving probability and statistics and items concerned with reasoning and judgment. At age 17, males held an advantage on each subset of items considered; this advantage was greatest on items concerned with geometry and measurement and nonroutine problems.

Differential course backgrounds of males and females have been cited as a possible cause of sex differences in mathematical performance. Forthcoming National Assessment documents will report on the extent to which course enrollment patterns can account for performance differences. In recent years, researchers have referred to a 2:1 ratio of males to females enrolled in the more advanced mathematics courses. The 1977–78 assessment found a 3:2 ratio of 17-year-old males to 17-year-old females who had taken advanced mathematics courses beyond Algebra II. Also, the percentage of the total student population taking such courses was not very large. For example, 5% of the 17-year-old males had taken Calculus or Precalculus compared with 3% of the females.

Differences between the sexes in enrollments were not found for lower-level courses including Algebra I, Geometry and Algebra II. It may be that differences in the mathematics enrollment patterns of the sexes are diminishing, and it is not likely that the differences that do exist can account for the total performance difference.

Race. At all three age levels, average achievement of whites (W) was higher than that of Hispanos (H) and blacks (B); Hispanic achievement tended to be higher than that of blacks. The differences between the three racial groups reported upon appear to increase with the age of the students (Exhibit 36). Approximately 11 percentage points separated achievement of whites and Hispanos at age 9; approximately 13 percentage points separated whites and blacks at this age. By age 17, whites and Hispanos differed by about 15 percentage points, while whites and blacks differed by about 21 percentage points.

There were some important differences between blacks and whites with respect to mathematics-course enrollment patterns, especially for lower-level academic mathematics courses. Three-fourths of 17-year-old whites had taken Algebra I compared with 55% of 17-year-old blacks. The percentages of 17-year-old whites and blacks who had taken Geometry were 55% and 31%, respectively, and for Algebra II, 38% and 24%, respectively.

Parental education. The relative position of the various parental-education groups remained constant at all three ages, with those whose parents had higher levels of education doing better than those whose parents had

EXHIBIT 35. Differences in Mean Percentages Between Sex Groups and the Nation on Application Items, Ages 9, 13 and 17

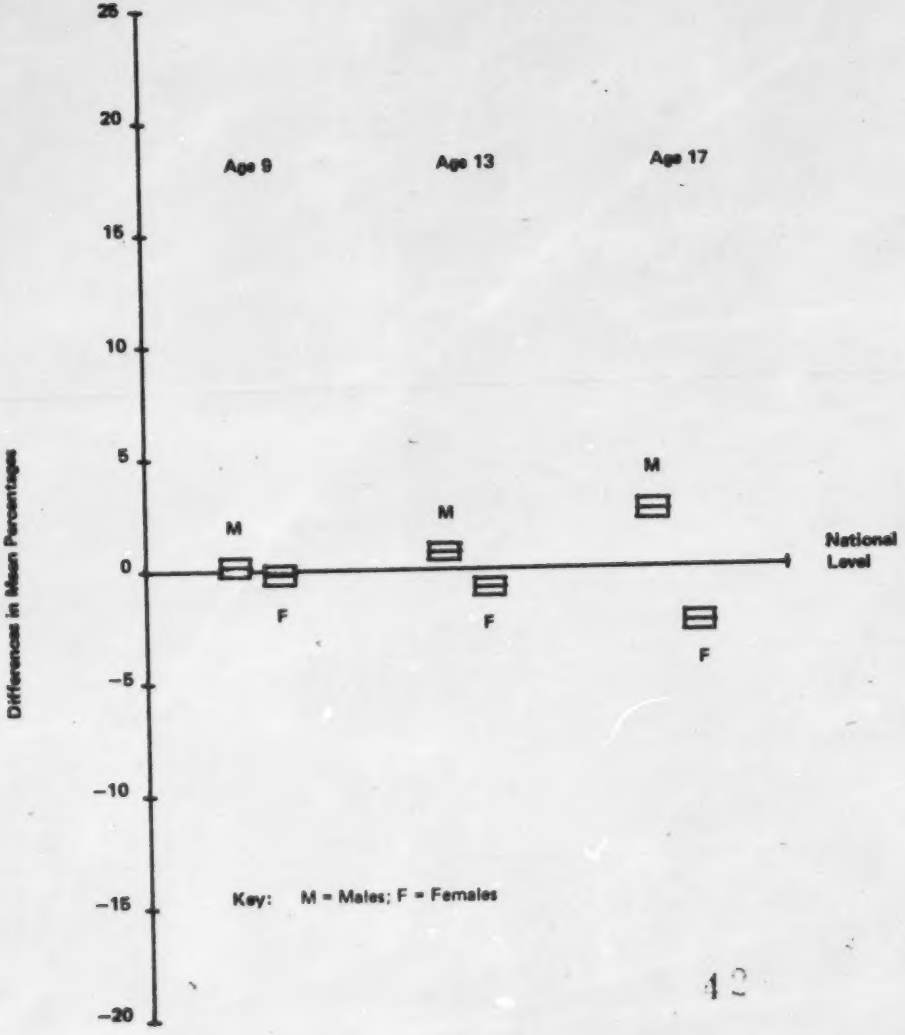


EXHIBIT 36. Differences in Mean Percentages Between Racial/Ethnic Groups and the Nation on Application Items, Ages 9, 13 and 17

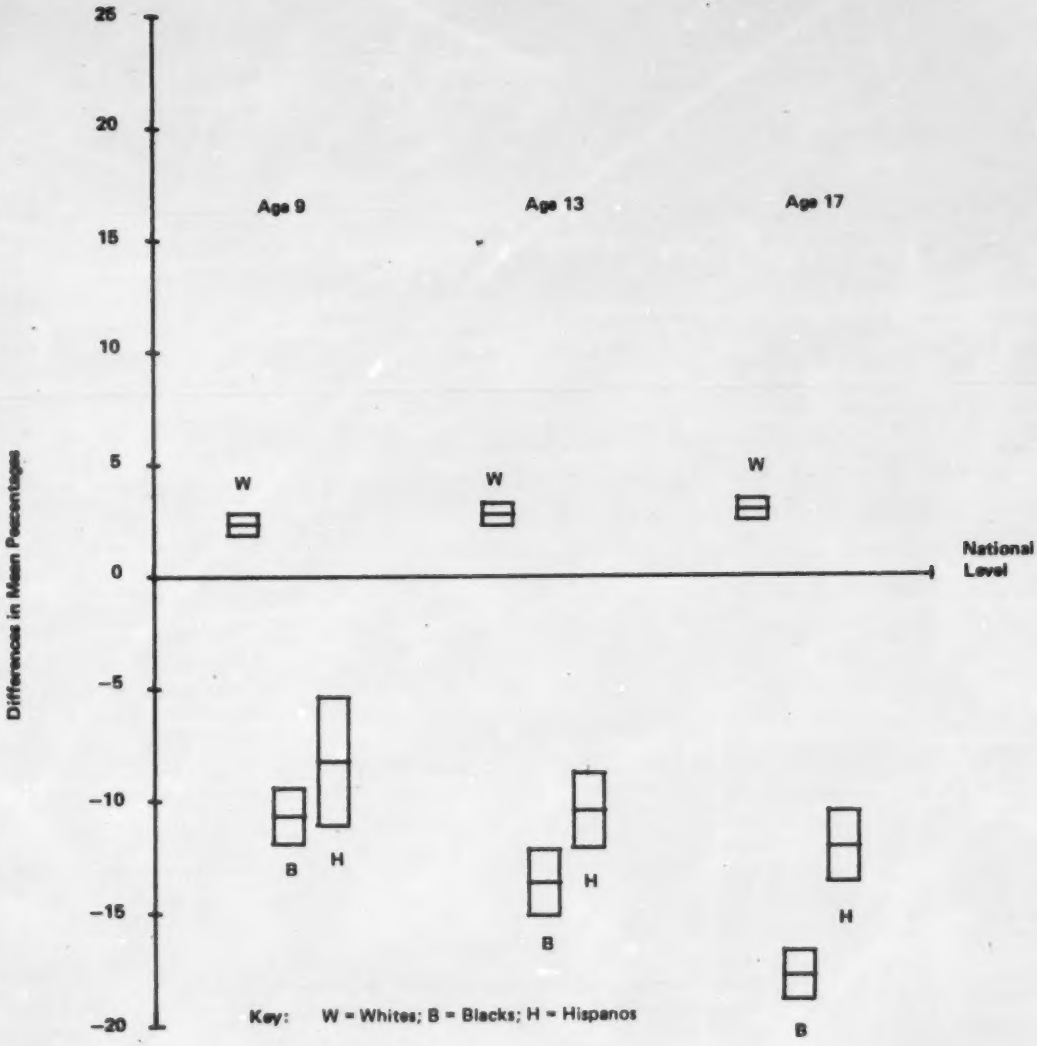
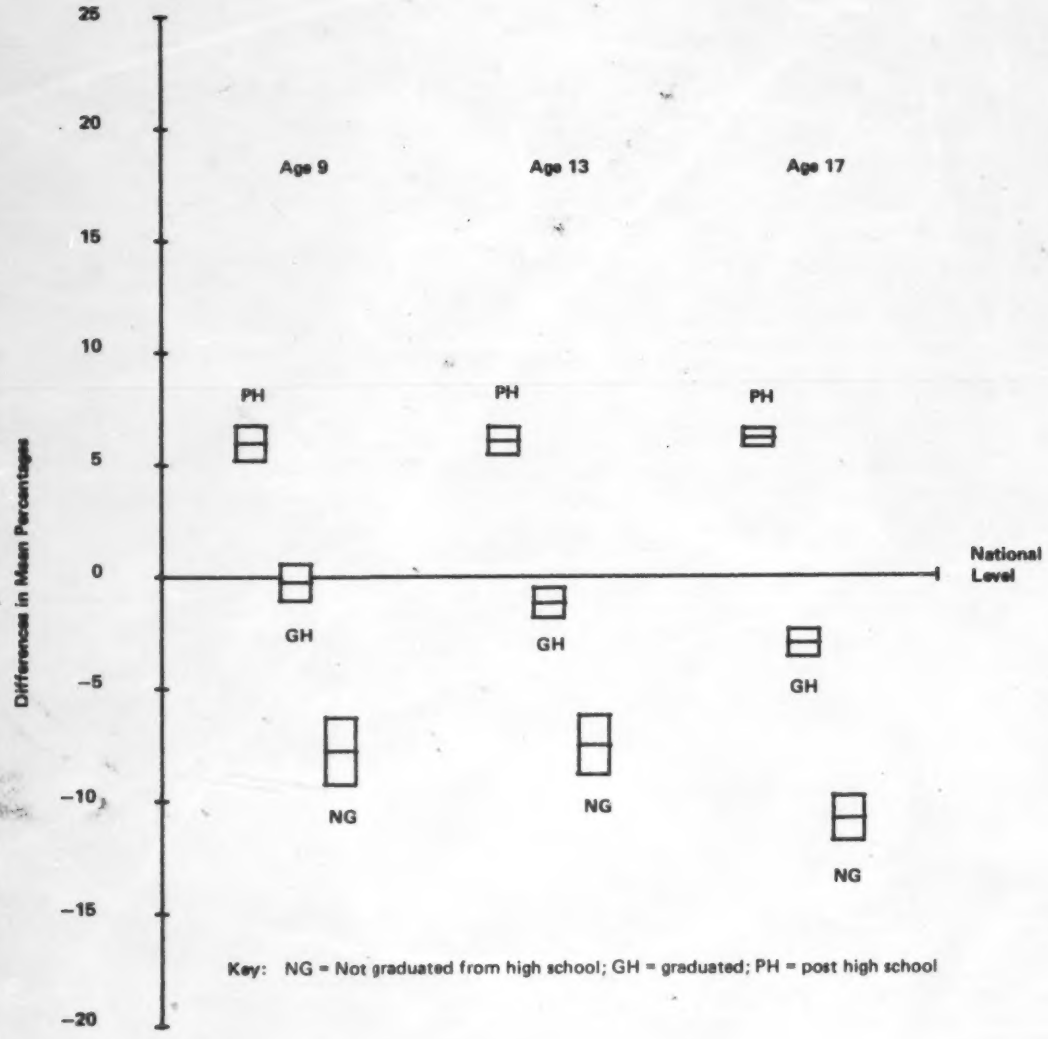


EXHIBIT 37. Differences in Mean Percentages Between Parental-Education Groups and the Nation on Application Items, Ages 9, 13 and 17



Key: NG = Not graduated from high school; GH = graduated; PH = post high school

EXHIBIT 38. Differences in Mean Percentages Between Type-of-Community Groups and the Nation on Application Items, Ages 9, 13 and 17

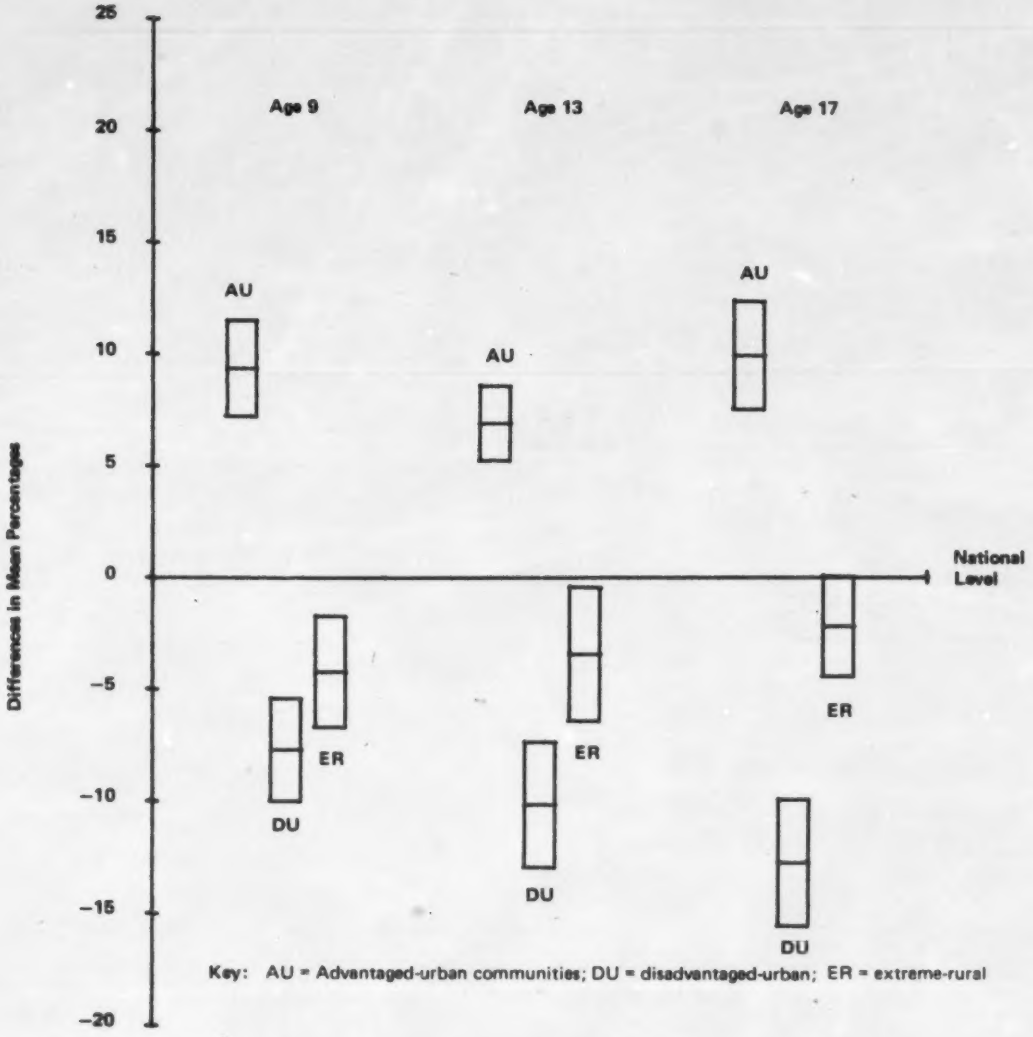
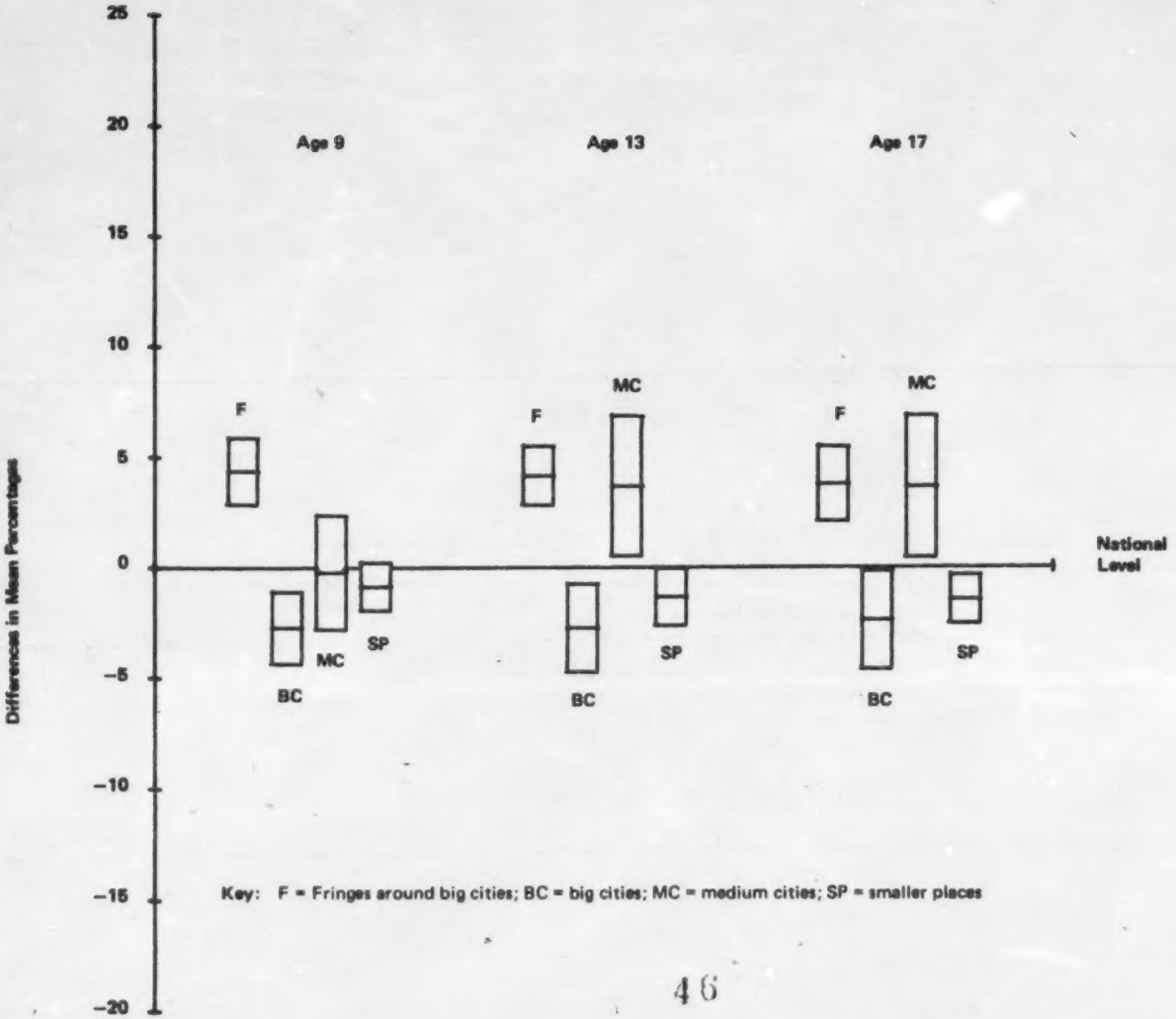
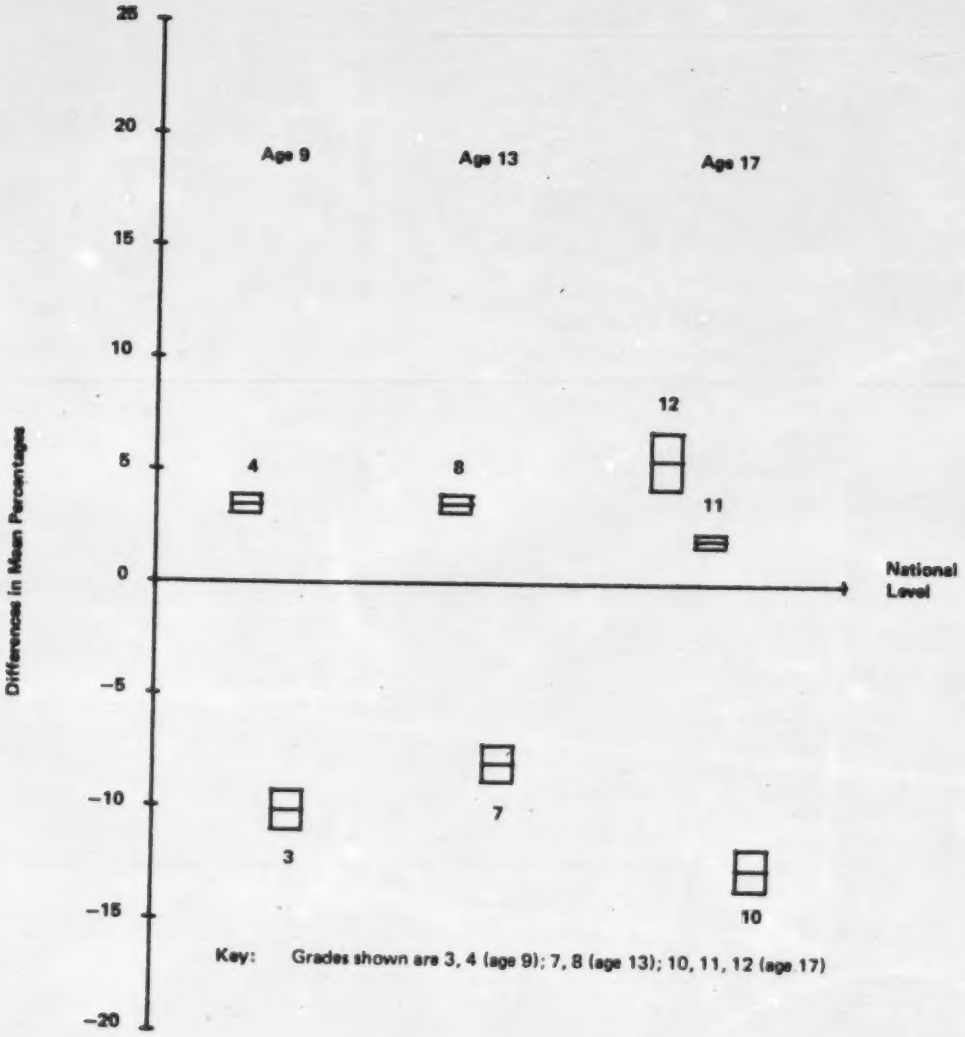


EXHIBIT 39. Differences in Mean Percentages Between Size-of-Community Groups and the Nation on Application Items, Ages 9, 13 and 17



Key: F = Fringes around big cities; BC = big cities; MC = medium cities; SP = smaller places

EXHIBIT 40. Differences in Mean Percentages Between Grade-in-School Groups and the Nation on Application Items, Ages 9, 13 and 17



less education. However, differences in performance tended to increase slightly by age 17 (Exhibit 37). Those with at least one parent who had some education beyond high school (PH) were above the national level at each age, while those whose parents had not graduated from high school (NG) were consistently below the nation. Those with at least one parent who had graduated from high school (GH), but had not continued their education were at the national level at age 9 but dropped successively farther below the nation at the older ages.

Type of community. National Assessment reports results for three extreme types of communities: advantaged urban (AU), disadvantaged urban (DU) and extreme rural (ER). As shown in Exhibit 38, the advantaged-urban groups were consistently above the national level and the disadvantaged-urban groups were consistently below it. Disadvantaged-urban students' achievement relative to the nation appeared to decline with age, while that of extreme-rural students tended to improve.

Size of community. Four groups are defined by National Assessment based solely on community size: fringes around big cities (F), big cities (BC), medium cities (MC) and smaller places (SP). The fringes around big cities were above the nation at each age, and big cities were below national levels (Exhibit 39). Medium cities, although showing achievement similar to the nation at age 9, were above the nation at ages 13 and 17. By age 17, smaller places were slightly below the national level.

School Variables

Grade in school. In the National Assessment sample, about three-fourths of the 9-year-olds were in the 4th grade (4) and one-fourth, the 3rd grade (3); about three-fourths of the 13-year-olds were in the 8th grade (8) and one-fourth, the 7th grade (7); and about 75% of the 17-year-olds were in the 11th grade (11), with about 15% in the 10th grade (10) and 10% in the 12th grade (12). As one would expect, those in a higher grade at each age level showed higher mathematics achievement than those in a lower grade (Exhibit 40).

Mathematics courses taken (17-year-olds). Seventeen-year-olds were asked to supply information on mathematics courses they had taken. Exhibit 41 shows the percentages who reported that they had taken various courses.

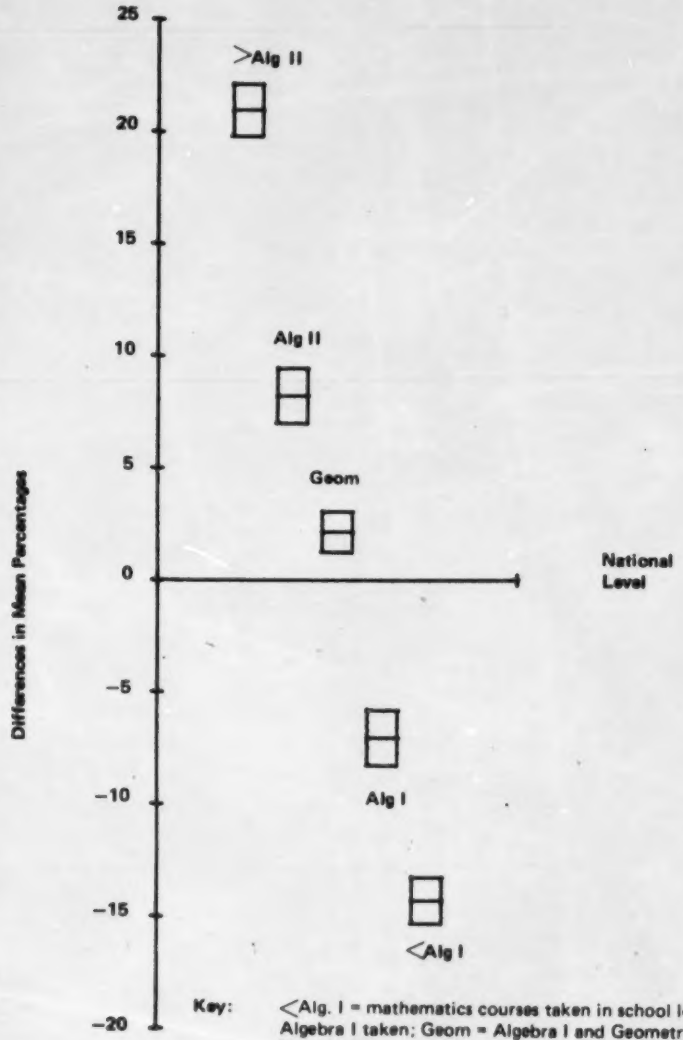
EXHIBIT 41. Percentages of 17-Year-Olds Taking Various Mathematics Courses

	Percent Who Had Taken Course for 1/2 or 1 Year
Algebra I	72
Geometry	51
Algebra II	37
Trigonometry	13
Probability and statistics	3
Pre-calculus	4

Mathematics achievement on application problems was analyzed for the following groups of 17-year-olds: (1) those who had not taken any of the courses listed above (<Alg I); (2) those who had taken Algebra I but no more advanced courses (Alg I); (3) those who had taken Geometry but not Algebra II or any more advanced courses (Geom); (4) those who had taken Algebra II but not Trigonometry, Probability and Statistics, or Pre-calculus (Alg II); (5) those who had taken Algebra II and Trigonometry or Probability and Statistics or Pre-calculus (>Alg II).

Exhibit 42 shows performance for the different levels of mathematics course-taking experiences. Sizable differences are apparent for the different levels of mathematics courses taken on all subsets of application items analyzed. Differences appear greatest for multistep problems and problems involving measurement.

EXHIBIT 42. Differences in Mean Percentages Between Groups That Have Taken Different Levels of Mathematics Courses and the Nation on Application Items, Age 17



Key: <Alg. I = mathematics courses taken in school lower than Algebra I; Alg I = at least Algebra I taken; Geom = Algebra I and Geometry taken; Alg II = Algebra II taken; >Alg II = courses taken to the level of Algebra II and above

Age-Level Comparisons

Comparisons of national and group average performance levels for different age groups are based on only those items taken by two age groups. Nine- and 13-year-olds responded to 33 identical application items; 13- and 17-year-olds answered 83 identical application items.

As might be expected, performance improved between ages 9 and 13 and between ages 13 and 17. This improvement was most dramatic between ages 9 and 13. The mean achievement level of 13-year-olds was 29 percentage points above that of 9-year-olds; the performance of 17-year-olds was 17 percentage points above that of 13-year-olds.

Exhibits 43 and 44 compare performance of selected reporting groups on application items for two age levels. As seen in Exhibit 43, 13-year-olds uniformly showed higher achievement in mathematical application than 9-year-olds. The same pattern does not hold for 13- and 17-year-olds, however (Exhibit 44). The assessment found that, on the average:

- A higher percentage of white 13-year-olds answered mathematics application questions correctly than did black 17-year-olds.
- Thirteen-year-olds in advantaged-urban communities did better than 17-year-olds in disadvantaged-urban communities.
- Thirteen-year-olds whose parents had some post-high-school education performed at about the same level as 17-year-olds whose parents did not graduate from high school.

The improvement between 9- and 13-year-olds was very similar for most reporting groups analyzed. Thirteen-year-old blacks, Hispanos and disadvantaged-urban residents were farther below the national level than their 9-year-old counterparts, meaning the improvement between ages 9 and 13 for these groups was less than the national improvement. Similarly, the improvement between ages 13 and 17 was very much the same for most reporting groups. However, at age 17, blacks, Hispanos, those living in disadvantaged-urban areas and those whose parents had not graduated from high school were farther below the nation than 13-year-olds in these groups, again meaning that these groups showed smaller improvements between ages 13 and 17 than did the nation as a whole.

These findings suggest that not only are the achievement levels of racial/ethnic minorities and disadvantaged students in this country below majority and advantaged groups, but that the deficit may be getting worse as students get older.

EXHIBIT 43. Average Percentages of Correct Response on Identical Sets of Items for Selected Reporting Groups, Ages 9 and 13 (Number of Items = 33)

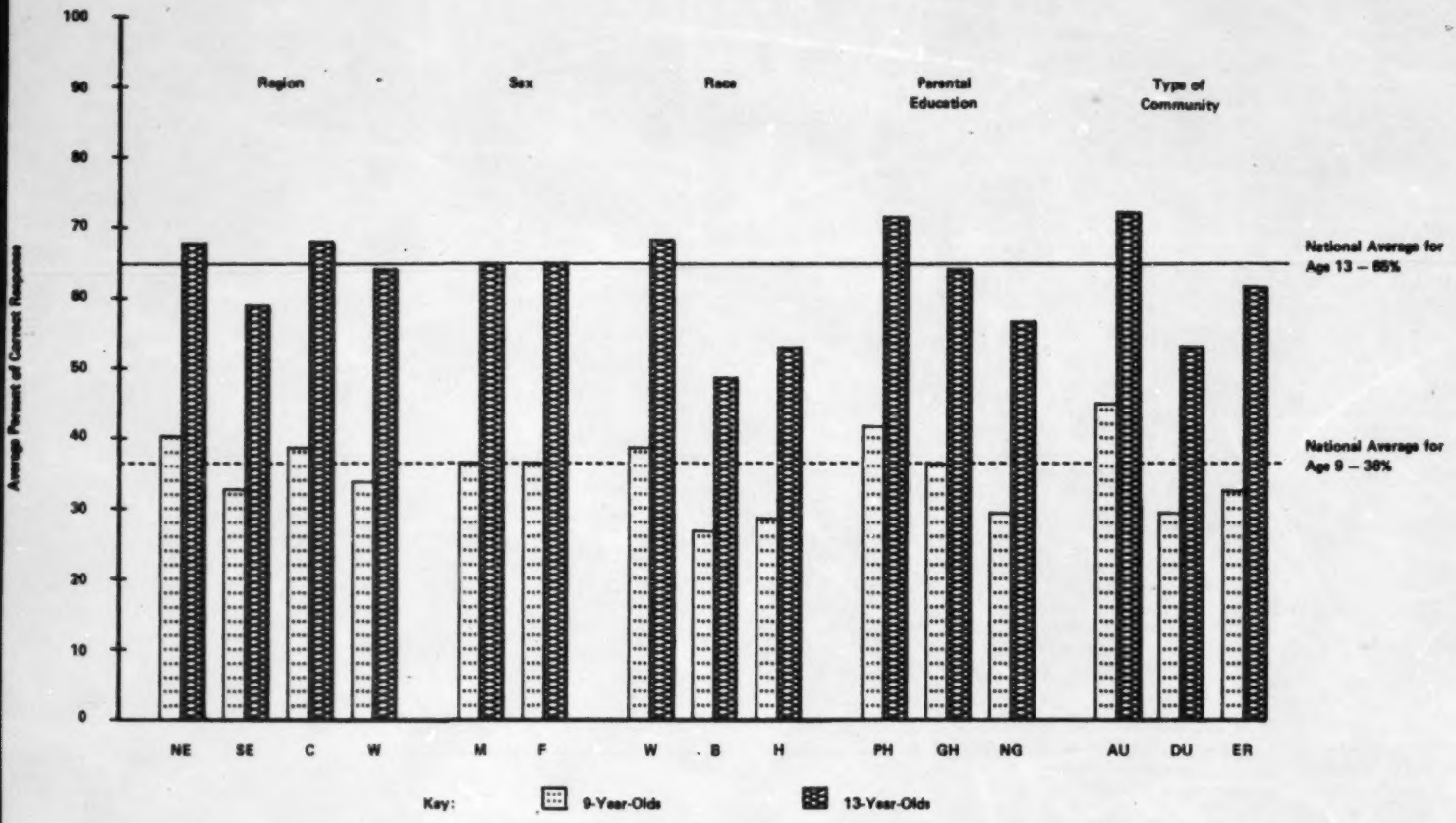
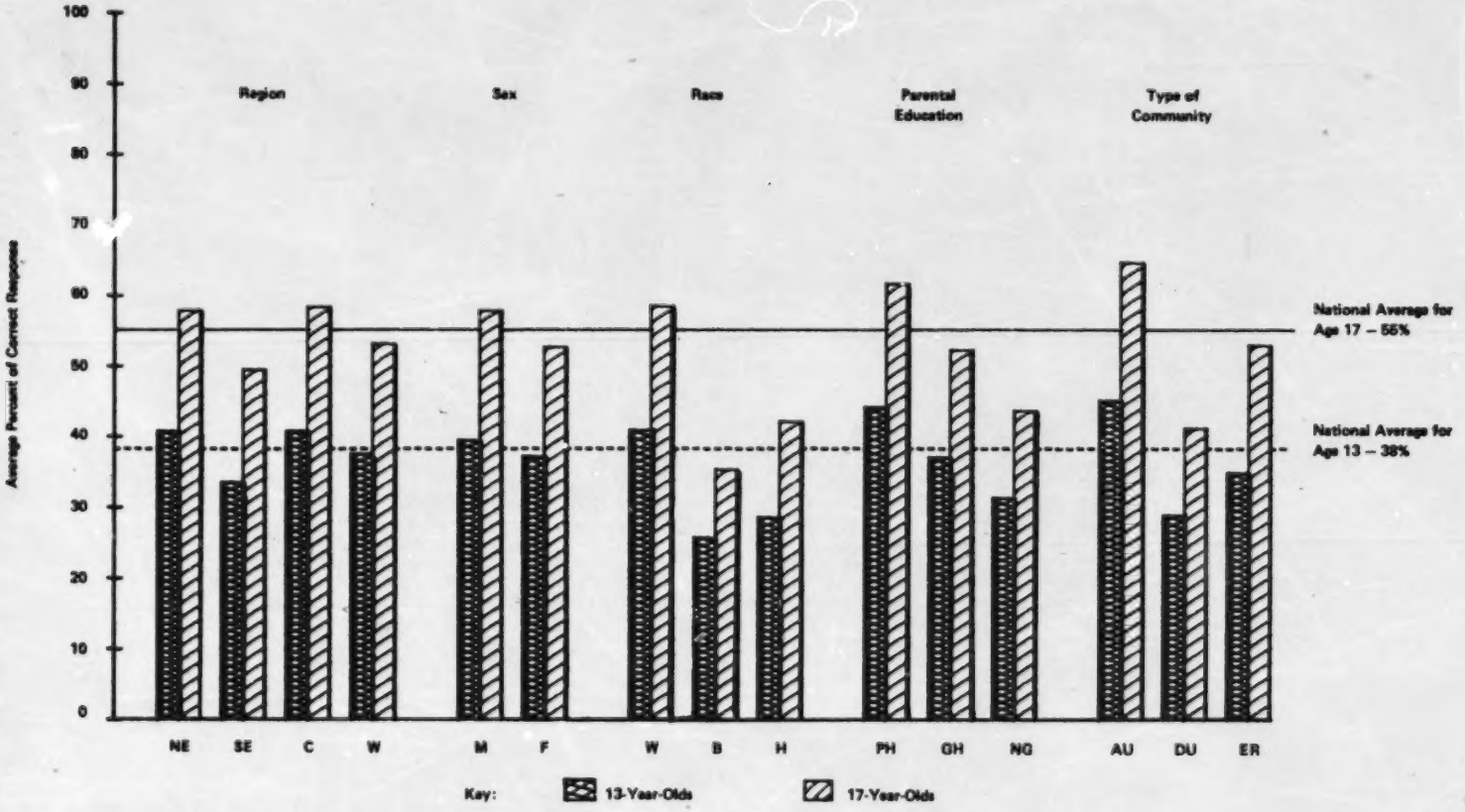


EXHIBIT 44. Average Percentages of Correct Response on Identical Sets of Items for Selected Reporting Groups, Ages 13 and 17 (Number of Items = 83)



CHAPTER 3

OBSERVATIONS ON THE RESULTS

The 1977–78 national assessment of mathematics provides a comprehensive survey of students' present levels of mathematics achievement. The extensive data described in this report are best interpreted in light of current expectations and trends in mathematics education. Accordingly, National Assessment convened a panel of persons active in the field to interpret the results and to consider the implications for mathematics education.

Panel members included: Iris Carl, mathematics resource center specialist and curriculum writer, Houston (Texas) Independent School District, and member of the National Council of Teachers of Mathematics (NCTM) Program and Instructional Affairs Committees; Thomas Carpenter, associate professor of curriculum and instruction, University of Wisconsin, Madison; Robert Dilworth, professor of mathematics, California Institute of Technology, Pasadena; Edgar Edwards Jr., associate director of mathematics (K-12), Virginia Department of Education, Richmond, and member of the NCTM Board of Directors; and James W. Wilson, professor and department head, Mathematics Education Department, University of Georgia, Athens, and member of the NCTM Board of Directors.

This chapter presents selected quotes and a summary of comments made during a two and one-half day meeting at which the panel informally reviewed and discussed the mathematics results. Observations of the panel members represent their opinions as individuals and do not necessarily reflect the positions of any organizations with which they are associated.

The panel considered results for the mathematics cognitive process levels of knowledge, skill and application in addition to data on changes in performance between 1973 and 1978. The observations presented here, although focusing on the application results, also reflect data found in other NAEP reports (*Mathematical Knowledge and Skills*, 1979; *Changes in Mathematical Achievement, 1973–78*, 1979).

Panel members emphasized cautions that must be kept in mind when interpreting the data. The format of and the vocabulary used in an item can have an influence on performance. For example, multiple-choice items are often easier to answer than open-ended items, even if the task is the same. Specific vocabulary may present difficulties, as in the case in which many more students could calculate the "distance around" a figure than could find the "perimeter" of the same figure. Thus, in generalizing from performance on an item to ability to perform a task, one must take into account the nature of the particular items used in the assessment. In addition, although students are told that the assessment is not a test and that their scores will not be recorded, the assessment is a timed, test-like situation, and responses given under the stress of the situation may not be totally accurate indicators of ability.

A Historical Perspective

Amid reports of declining test scores and public disenchantment with curricular and teaching innovations, there has been widespread demand during the 1970s that educa-

tion go "back to the basics." A number of states and local districts have instituted "minimal-competency" requirements for high school graduates, and schools increasingly have been held "accountable" for the performance of their students.

In mathematics, the movement to the "basics" has often resulted in a narrowing of the curriculum, stressing computational skills and knowledge of facts and definitions with less time available for instruction in other topics. Wilson noted that textbooks have changed during the 1970s: "By the mid-1970s, textbooks in the schools emphasized computational skills. Word problems were streamlined to present students with the least possible amount of complexity."

The increased testing associated with minimal competency and accountability movements may encourage teachers to emphasize material found in tests and to devote less effort to material that is not generally tested. Teachers also may be inclined to stress areas that can be fairly easily impacted by increased classroom effort — namely lower-level cognitive processes such as knowledge and simple skills.

Training of mathematics teachers is not consistent across the country, and some persons teaching mathematics may not receive sufficient training. For example, elementary teachers may not be required to take any mathematics beyond what they have had in school. This may lead to a fear of teaching mathematics. Dilworth described a group of elementary teachers at workshops held at the Teacher Center in San Diego, California: "Out of 65 teachers, many indicated that prior to the workshops they felt very insecure in teaching mathematics. When interviewed at the Center following the workshops, they all felt that they could teach mathematics with confidence. But, what happens to the teachers who do not attend such workshops?" Edwards stated that many small colleges do not offer teaching methods courses specific to mathematics. He added that "too often, new teachers are simply given a textbook and no guidance on effective teaching methods."

Implications of the Results

How well can students solve problems? The panel felt that problem solving is a major area of concern in mathematics education. Carpenter stated, "Children's performance on textbook-like, simple, one-step word problems is pretty good. It's not as good as performance on computational exercises using the same numbers, but that's to be expected since a word problem adds another level of complexity. However, on more complex problems or on problems that deviate even slightly from the textbook format, children have difficulty. It's apparent that many students are not adept at extracting the meaning of a problem situation."

Students do not seem to "think through" the problems presented. For example, on several skill- and understanding-level items, many more students understood that the sides of a square are equal and were able to find the area of a rectangular figure given length and width than were able to apply those two pieces of information to find the area of a square when only one side was given.

Dilworth noted that extraneous information in a problem appeared to be very difficult for students to cope with. Only 56% of the 13-year-olds answered the following multiple-choice problem correctly, although the multiplication required (5 × 2) is extremely simple.

One rabbit eats 2 pounds of food each week. There are 52 weeks in a year.

How much food will 5 rabbits eat in one week?

Answers selected indicate that many 13-year-olds randomly multiplied numbers together.

Many 17-year-olds had difficulty in figuring out the unit price for kilowatt hours consumed when shown a sample electric bill. Approximately 10% set up the problem and

divided correctly; another 7% set up the problem correctly but made an error in division. The problem involves only one step, but it seems that the extra figures given may have confused many students.

Edwards noted that a problem of this nature would not normally be encountered in students' textbooks. He questioned the level of performance to be expected on types of problems with which students are not familiar and observed that vocabulary can be a barrier to problem solving.

The panel concluded that students should be able to deal with a variety of problems presented in different contexts. However, this does not seem to be the case at present.

The textbooks that have come into widespread popular use during the 1970s may have contributed to difficulties with problem solving. Problems that do appear in the texts are often very homogeneous, with items requiring addition at the end of a unit on addition, subtraction word problems at the end of a subtraction unit, and so forth. Many problems are "streamlined," or presented with as few words as possible — for example, "52 marbles. 17 taken away. How many left?" Edwards stated that "children may have difficulty with vocabulary even though they have the skills to deal with a problem." However, the panel felt that potential vocabulary difficulties should not result in the simplification of *all* word problems. Wilson stated, "If reading is a difficulty in children's mathematical performance, it is irresponsible of us to protect them from reading."

Textbooks often promote a mechanistic approach to problem solving, encouraging students to look for "key words" or clues in a problem to decide which operation to use, rather than trying to extract the meaning of a problem situation. If the student gets confused and forgets which algorithm fits which key word, or if a problem is stated in a way that is out of the ordinary, the student may well be at a loss in solving problems. As Carpenter said, "There has been an attempt to take any kind of thinking out of the problem-solving process. The shifts in curriculum have made it easy for children not to think."

Often students are not allowed to go on to higher-level processes until they have completely mastered lower-level ones. The panel felt that this could severely limit students' exposure to a wide range of mathematical topics and could result in students having very little idea of how the various things they had learned were related. For example, data reported in *Mathematical Knowledge and Skills* (1979) showed that considerably more students added fractions successfully than correctly estimated the answer to 7/8 + 12/13, indicating that many do not understand the concept of the magnitude of fractions. Carl noted that in the Houston (Texas) schools, programs that require children to move at a set speed often produce better results than those that insist on complete mastery of one topic before moving on to another.

The increased emphasis on testing occurring in recent years has also had an effect on the curriculum and on teaching practices. Teachers may tend to emphasize topics and skills that will appear in the tests. Lower-level skills and knowledge are often easier to test, and it is generally easier to bring about changes in these areas. Carl noted that in one state, which has instituted a minimal-competency testing program, in some cases the test has become the curriculum and children are not exposed to a comprehensive mathematics program.

As noted previously, teacher education in methods of teaching mathematics may be less than adequate, and teachers may not be well prepared to teach problem solving. Wilson stated that "people seem to think that by emphasizing knowledge and skills, they have done enough. They seem to feel that facility with lower-level processes should automatically transfer into an ability to solve problems, and that is not necessarily the case." Teachers may not know effective ways to approach problem solving and may fall back on memorization and rote drill as an

easy alternative. Wilson shared his finding that, in his experiences in schools, many teachers in the schools he visited simply did not conduct lessons on problem solving.

Teaching children to think through problems is a difficult task for which there are no "sure" methodologies. Furthermore, efforts to teach problem solving do not always produce easily observable or measurable results. However, the panel felt strongly that current emphasis on selecting an algorithm in a mechanical way and getting the right answer rather than understanding the concepts involved is not in the best interests of the students. Carpenter stated that "we need to make it clear that the ability to analyze a problem situation is equally as important as getting the right answer."

Difficulties in problem solving were seen as affecting most of the problem topics discussed in this report — consumerism, percent, measurement, probability and statistics. Panel members emphasized very strongly that additional drill in solving problems mechanically was *not* recommended. Wilson stated, "What we math educators have been doing in the problem-solving area does not appear to be working." Dilworth concurred, saying that "new approaches are needed to improve problem-solving performance." The panel did feel that a greater variety of problems should be included in textbooks. As Wilson noted, "What is not in textbooks often is not taught. It would be better to include a variety of problems, so that those who wanted to could use them, rather than leaving out some types of problems all together. At least some more exposure to difficult problem situations would be provided."

In examining results for population groups, panel members expressed some surprise that differences between various groups were similar for each of the four cognitive process levels assessed (knowledge, skill, understanding and application). The only exception to this pattern is for 17-year-old males and females. At this age, males did somewhat better than females on application items, while their performance was more similar on knowledge and skill items.

The interpretive panel felt that many factors have an influence on differences in group performance and that the information available was not sufficient to single out particular ones. They did feel, however, that the data present evidence that inequities in American education have not been eliminated.

Recommendations

In light of the data reported by NAEP, the panel felt that a number of recommendations are in order:

1. An expanded definition of what is "basic" in mathematics is crucial to foster students' ability to cope with different types of mathematical problems. Students must be introduced to exercises involving higher-level, as well as lower-level, cognitive processes.

2. Textbooks should be modified to include a greater variety of problem-solving tasks. Approaches to problem solving should not be of the type that encourage mechanistic, algorithmic methods. It should be emphasized that the ability to analyze a problem situation is equally as important as the correct solution.

3. Teaching of problem solving should receive more emphasis in the schools. It appears that many teachers feel pressure to drill for mastery of skills and not to teach problem solving. Additional drill on an expanded number of topics should *not* be encouraged.

4. Success on tests stressing low-level cognitive processes should not be the only criterion used to measure the effectiveness of mathematics programs. Too often the test then becomes the objective, and important

areas that require more time and effort to teach fall by the wayside.

5. There is a need for more consistent and comprehensive teacher education in the area of mathematics. More attention should be devoted to preparing teachers to teach concepts and problem solving so they will not tend to rely as heavily on drill and memorization work.

6. Further research on methods of teaching problem solving is necessary so that teachers can approach it with some confidence that the strategies that they are using will be effective.

7. There is a need for further research into causes and potential remedies for the disparities in group performance that presently exist. Additional analysis planned by National Assessment, such as finer group breakdowns (sex by race, region by race, and so on), and information on differential course-taking behavior will provide useful data that should be carefully studied.

APPENDIX A

DEFINITIONS OF REPORTING GROUPS

National Assessment examines results for various groups of people within the national population. These groups are defined as follows.

Background Variables

Region

The country has been divided into four regions: Northeast, Southeast, Central and West. States included in each region are shown on the following map.

Sex

Results are reported for males and females.

Race

Results are presented for blacks, whites and Hispanos.

Level of Parental Education

Three categories of parental-education levels are defined by National Assessment, based on students' reports. These categories are: those whose parents did not graduate from high school, those who have at least one parent who graduated from high school and those who have at least one parent who has had some post-high-school education.

Type of Community

Communities in this category are defined by an occupational profile of the area served by a school as well as by the size of the community in which the school is located.

Advantaged-urban communities. Students in this group attend schools in or around cities having a population greater than 200,000 where a high proportion of the residents are in professional or managerial positions.

Disadvantaged-urban communities. Students in this group attend schools in or around cities having a population greater than 250,000 where a high proportion of the residents are on welfare or are not regularly employed.

Extreme-rural communities. Students in this group attend schools in areas with a population under 10,000 where most of the residents are farmers or farm workers.

Size of Community

Big cities. Students in this group attend

schools within the city limits of cities having a 1970 census population over 200,000.

Fringes around big cities. Students in this group attend schools within metropolitan areas (1970 U.S. Bureau of the Census urbanized areas) served by cities having a population greater than 200,000 but outside the city limits.

Medium cities. Students in this group attend schools in cities having a population between 25,000 and 200,000, not classified in the fringes-around-big-cities category.

Smaller places. Students in this group attend schools in communities having a population less than 25,000, not classified in the fringes-around-big-cities category.

School Variables

Results are categorized for 9-year-olds in the 3rd or 4th grade, 13-year-olds in the 7th or 8th grade, and 17-year-olds in the 10th, 11th or 12th grade.

Mathematics Courses Taken by 17-Year-Olds

Seventeen-year-olds were asked about their mathematics course-taking experiences. A student was said to have taken a course if he or she had taken it for one-half year or longer. Five levels of mathematics-course experience are defined: (1) those who had not taken Algebra or any more advanced mathematics courses; (2) those who had taken Algebra I but no more advanced courses; (3) those who had taken Geometry but not Algebra II or any more advanced courses; (4) those who had taken Algebra II but no more advanced courses (Trigonometry, Probability and Statistics, or Pre-calculus); (5) those who had taken Algebra II and at least one more advanced course (Trigonometry, Probability and Statistics, or Pre-calculus).

APPENDIX B

GROUP DIFFERENCES FROM THE NATION FOR VARIOUS SETS OF MATHEMATICS APPLICATION ITEMS

EXHIBIT 45. Group Differences From the Nation for Various Sets of Mathematics Application Items, Age 9

	All Application Items	One-Step Word Problems	Consumer Problems
Number of items	44	18	7
National percentage	37.7	44.4	32.7
Region			
Northeast	3.9*	4.8*	2.2*
Southeast	- 4.0*	- 5.2*	- 3.6*
Central	2.3*	2.7*	3.1*
West	- 2.6*	- 2.8*	- 2.1*
Sex			
Male	0.2	- 0.1	- 0.2
Female	- 0.2	0.1	0.1
Race			
White	2.4*	3.0*	1.9*
Black	-10.7*	-14.3*	- 9.8*
Hispanic	- 8.2*	- 8.8*	- 5.9*
Parental education			
Post high school	6.0*	6.9*	5.3*
Graduated high school	- 0.2	0.3	1.2
Not graduated high school	- 7.8*	-11.0*	- 9.4*
Type of community			
Advantaged urban	9.4*	12.5*	7.1*
Disadvantaged urban	- 7.7*	-11.0*	- 7.3*
Extreme rural	- 4.2*	- 5.7*	- 3.4*
Size of community			
Fringes around big cities	4.4*	5.4*	3.4*
Big cities	- 2.7*	- 3.4*	- 2.8*
Medium cities	- 0.2	- 1.1	- 1.8
Smaller places	- 0.9	- 0.8	- 0.1
Grade in school			
3	-10.2*	-13.5*	- 8.7*
4	3.5*	4.6*	3.0*

**Indicates mean percentages significantly different from the nation at the .05 level.*

EXHIBIT 46. Group Differences From the Nation for Various Sets of Mathematics Application Items, Age 13

	All Application Items	One-Step Word Problems	Consumer Problems	Multi-step Problems	Graph and Table Problems	Geometry Problems	Probability and Statistics Problems	Reasoning and Judgment Problems
Number of items	106	27	20	13	13	13	11	13
National percentage	43.3	52.1	44.5	36.8	55.3	25.9	32.5	61.2
Region								
Northeast	2.7*	4.6*	3.0*	2.2*	3.5*	1.2	2.1*	1.3*
Southeast	- 5.1*	- 6.8*	- 5.2*	- 4.3*	- 7.7*	- 2.3*	- 4.0*	- 4.5*
Central	2.6*	2.8*	3.2*	3.2*	3.7*	0.6	2.3*	2.2*
West	- 0.9	- 1.3	- 1.6	- 1.9	- 0.5	0.3	- 1.0	0.2
Sex								
Male	0.8*	0.2	0.2	1.7*	1.4*	0.9*	0.2	0.1
Female	- 0.8*	- 0.3	- 0.3	- 1.7*	- 1.5*	- 1.0*	- 0.2	- 0.1
Race								
White	2.8*	3.6*	3.1*	2.7*	3.9*	1.2*	2.4*	2.4*
Black	-13.7*	-18.4*	-14.9*	-12.6*	-19.6*	- 6.3*	-10.8*	-11.0*
Hispanic	-10.5*	-12.6*	-12.2*	-11.6*	-13.5*	- 3.8*	-11.0*	- 9.9*
Parental education								
Post high school	6.1*	7.7*	6.9*	6.8*	8.5*	3.0*	4.6*	4.8*
Graduated high school	- 1.2*	- 1.1*	- 1.2*	- 1.6*	- 1.5	- 1.1*	- 0.8*	- 0.9*
Not graduated high school	- 7.5*	- 9.3*	- 8.3*	- 7.8*	-11.5*	- 3.3*	- 7.5*	- 4.8*
Type of community								
Advantaged urban	6.9*	7.5*	7.5*	8.9*	9.5*	4.5*	7.3*	4.6*
Disadvantaged urban	-10.2*	-12.4*	-12.0*	-11.4*	-16.2*	- 4.8*	- 7.4*	- 7.3*
Extreme rural	- 3.4*	- 4.1	- 3.4*	- 4.0*	- 5.5*	- 1.1	- 3.8*	- 2.7*
Size of community								
Fringes around big cities	4.2*	4.6*	4.8*	4.3*	6.5*	2.3*	4.1*	3.9*
Big cities	- 2.7*	- 3.9*	- 2.8*	- 1.6	- 3.9*	- 1.5	- 1.6	- 2.6*
Medium cities	3.7*	4.9*	3.7	4.2	4.7	2.7*	2.0	3.1*
Smaller places	- 1.4*	- 1.2	- 1.6*	- 2.0*	- 2.1*	- 0.9	- 1.8*	- 1.2*
Grade in school								
7	- 8.0*	-11.1*	- 9.6*	- 9.2*	-10.2*	- 4.0*	- 6.2*	- 5.7*
8	3.5*	5.0*	4.2*	4.0*	4.8*	1.6*	2.5*	2.5*

**Indicates mean percentages significantly different from the nation at the .05 level.*

EXHIBIT 47. Group Differences From the Nation for Various Sets of Mathematics Application Items, Age 17

	All Application Items	One-Step Word Problems	Consumer Problems	Multi-step Problems	Graph and Table Problems	Geometry Problems	Measure-ment Problems	Perimeter, Area, Volume Problems	Probability and Statistics Problems	Reasoning and Judgment Problems	Non-routine Problems
Number of items	136	20	33	30	15	18	13	9	13	12	10
National percentage	43.5	57.1	37.0	35.1	63.2	36.8	33.1	26.2	25.5	64.4	40.5
Region											
Northeast	2.6*	3.2*	1.9	2.8*	2.9*	2.3*	2.8*	3.3*	1.6	1.1	2.3
Southeast	-5.2*	-6.7*	-4.5*	-4.1*	-7.4*	-5.1*	-6.8*	-6.8*	-3.6*	-3.0*	-5.3*
Central	3.1*	3.8*	3.1*	2.7*	3.7*	3.4*	4.2*	4.1*	2.5*	2.2*	2.9*
West	-2.1*	-2.5*	-2.0*	-2.5*	-1.6	-2.4*	-2.3*	-2.6*	-1.6*	-1.5	-1.6
Sex											
Male	2.6*	2.6*	2.4*	2.5*	1.7*	3.3*	3.9*	4.4*	2.1*	1.9*	3.3*
Female	-2.5*	-2.5*	-2.2*	-2.4*	-1.6*	-3.1*	-3.7*	-4.2*	-2.0*	-1.8*	-3.2*
Race											
White	3.0*	3.6*	2.9*	3.2*	3.9*	2.5*	3.2*	2.8*	2.0*	2.1*	2.9*
Black	-17.8*	-21.7*	-17.7*	-19.2*	-22.3*	-15.9*	-19.7*	-17.4*	-12.0*	-11.8*	-17.1*
Hispanic	-12.1*	-13.2*	-11.0*	-12.6*	-16.9*	-11.5*	-12.2*	-12.5*	-6.9*	-9.7*	-12.5*
Parental education											
Post high school	6.2*	6.4*	5.3*	6.5*	6.8*	6.9*	7.3*	7.5*	4.3*	3.8*	6.2*
Graduated high school	-3.0*	-2.9*	-2.4*	-3.3*	-2.5*	-3.8*	-3.7*	-4.3*	-2.3*	-2.0*	-2.8*
Not graduated high school	-10.8*	-10.9*	-9.7*	-11.7*	-12.3*	-10.9*	-12.8*	-12.3*	-7.4*	-6.4*	-12.2*

**Indicates mean percentages significantly different from the nation at the .05 level.*

EXHIBIT 47 *(Continued).* **Group Differences From the Nation for Various Sets of Mathematics Application Items, Age 17**

	All Application Items	One-Step Word Problems	Consumer Problems	Multi-step Problems	Graph and Table Problems	Geometry Problems	Measure-ment Problems	Perimeter, Area, Volume Problems	Probability and Statistics Problems	Reasoning and Judgment Problems	Non-routine Problems
Type of community											
Advantaged urban	9.9*	8.8*	9.0*	11.0*	9.4*	11.0*	13.8*	16.1*	8.6*	5.4*	9.6*
Disadvantaged urban	-12.8*	-15.6*	-11.8*	-13.5*	-13.0*	-12.8*	-15.5*	-14.2*	- 8.0*	- 9.4*	-11.8*
Extreme rural	- 2.2	- 1.8	- 0.7	- 1.2	- 3.5	- 3.2*	- 2.0	- 2.9	- 1.0	- 1.9*	- 2.7
Size of community											
Fringes around big cities	3.8*	3.2*	3.2*	3.5*	5.4*	5.2*	4.6*	4.4*	2.6*	1.6*	3.4*
Big cities	- 2.4*	- 3.9*	- 2.6*	- 3.0*	- 3.7*	- 1.6	- 2.4	- 1.5	- 0.7	- 1.3	- 1.9
Medium cities	3.6*	5.7*	3.9*	5.2*	2.5	3.1	3.8	2.8	3.0*	1.9	- 0.1
Smaller places	- 1.5*	- 0.8	- 1.0	- 1.4*	- 1.5	- 2.5*	- 1.9*	- 2.2*	- 1.6*	- 0.6	- 0.7
Grade in school											
10	-12.7*	-15.6*	-12.3*	-13.7*	-15.5*	-11.5*	-13.0*	-12.1*	- 8.2*	- 9.1*	-11.5*
11	2.0*	2.6*	1.7*	2.0*	2.5*	1.8*	1.8*	1.7*	1.1*	1.5*	1.9*
12	5.5*	5.1*	6.6*	6.6*	7.4*	4.6*	6.8*	6.4*	3.9*	3.3*	3.6*
Courses taken											
<Algebra I	-14.3*	-14.5*	-12.6*	-16.1*	-15.7*	-15.4*	-16.2*	-17.0*	- 8.8*	- 9.0*	-13.2*
Algebra I	- 7.1*	- 4.7*	- 5.2*	- 7.9*	- 4.4*	-12.0*	- 8.8*	-10.2*	- 4.4*	- 3.9*	- 6.9*
Geometry	2.2*	2.8*	1.0	1.0	5.0*	5.5*	- 0.2	- 0.8	- 1.0	1.9*	1.7
Algebra II	8.2*	8.6*	6.8*	9.0*	8.9*	8.7*	9.6*	9.5*	4.7*	5.8*	7.3*
>Algebra II	21.0*	18.4*	19.8*	25.5*	16.0*	24.9*	26.6*	30.5*	15.8*	11.4*	21.4*

**Indicates mean percentages significantly different from the nation at the .05 level.*

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